

10.4 Translating Sine and Cosine Graphs

Note: It says “10.3 Period and Phase Shift for Sine and Cosine” in your copy of the notes

$$y = a \sin(b(\theta - h)) + k$$

Vertical stretch

Amplitude
distance from
middle to
top / bottom

* flip if $a < 0$

Frequency

$$\text{Period} = \frac{2\pi}{b}$$

→ # of cycles
every 2π

Phase Shift
(horizontal
shift)

+ : left
- : Right

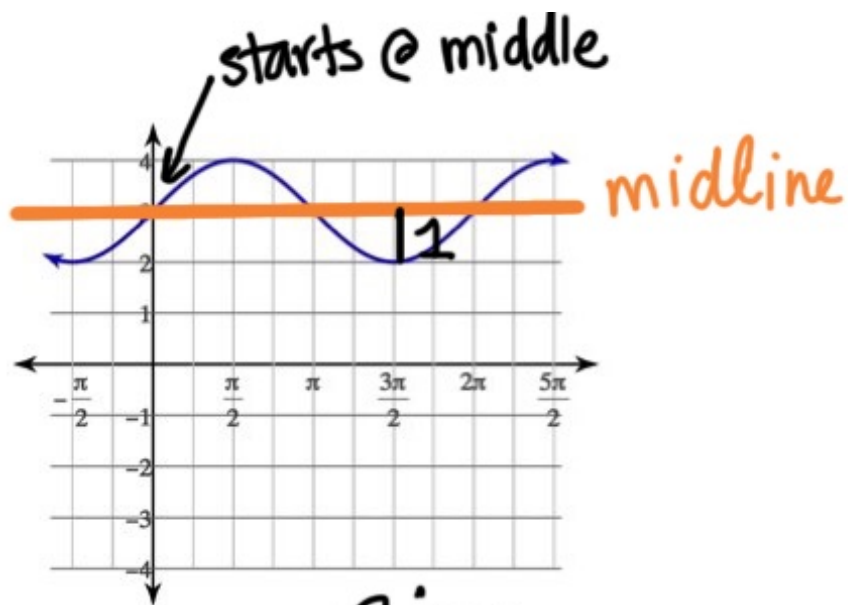
Midline

$$y = k$$

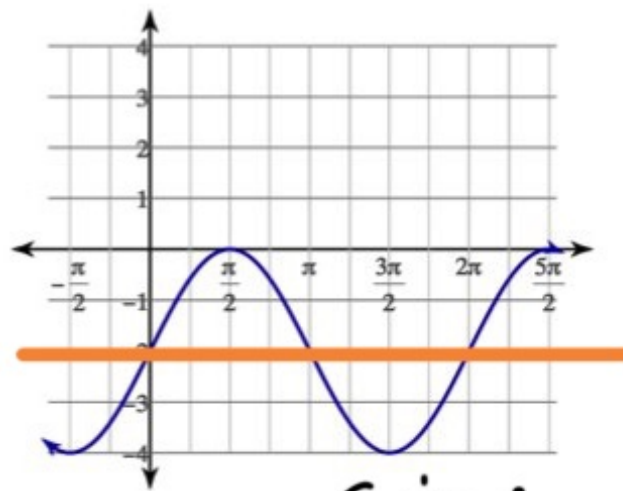
Vertical shift

+ : up
- : down

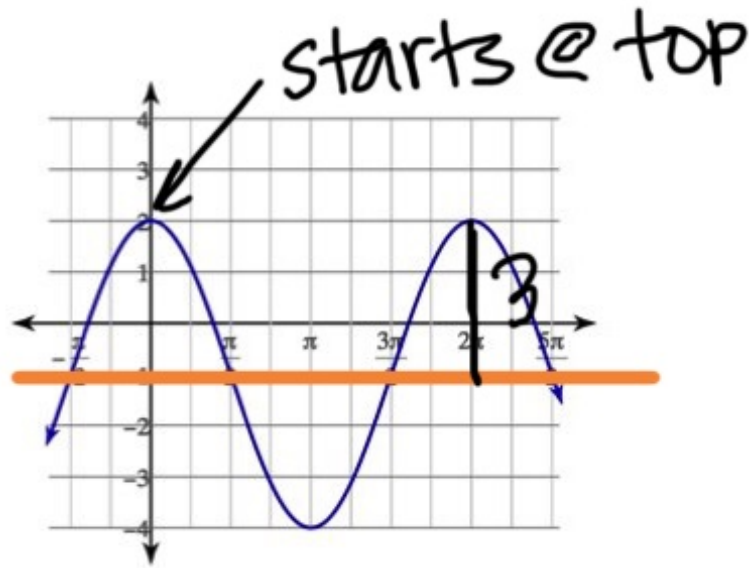
A.

Sine/Cosine? SineAmplitude: 1Midline: $y = 3$ Equation: $y = 1\sin\theta + 3$

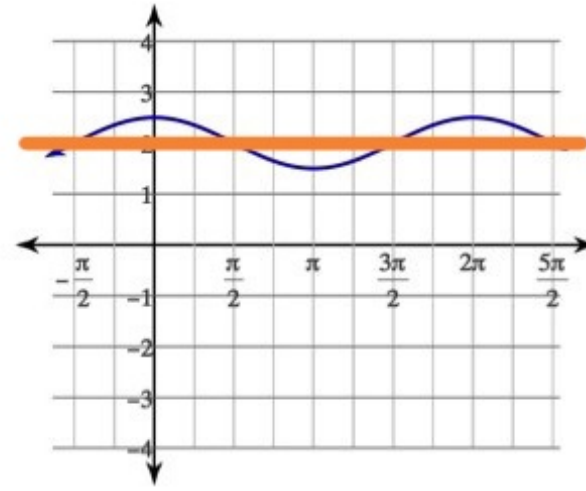
B.

Sine/Cosine? SineAmplitude: 2Midline: $y = -2$ Equation: $y = 2\sin\theta - 2$

C.

Sine/Cosine? cosineAmplitude: 3Midline: $y = -1$ Equation: $y = 3 \cos \theta - 1$

D.

Sine/Cosine? cosineAmplitude: $\frac{1}{2}$ Midline: $y = 2$ Equation: $y = \frac{1}{2} \cos \theta + 2$

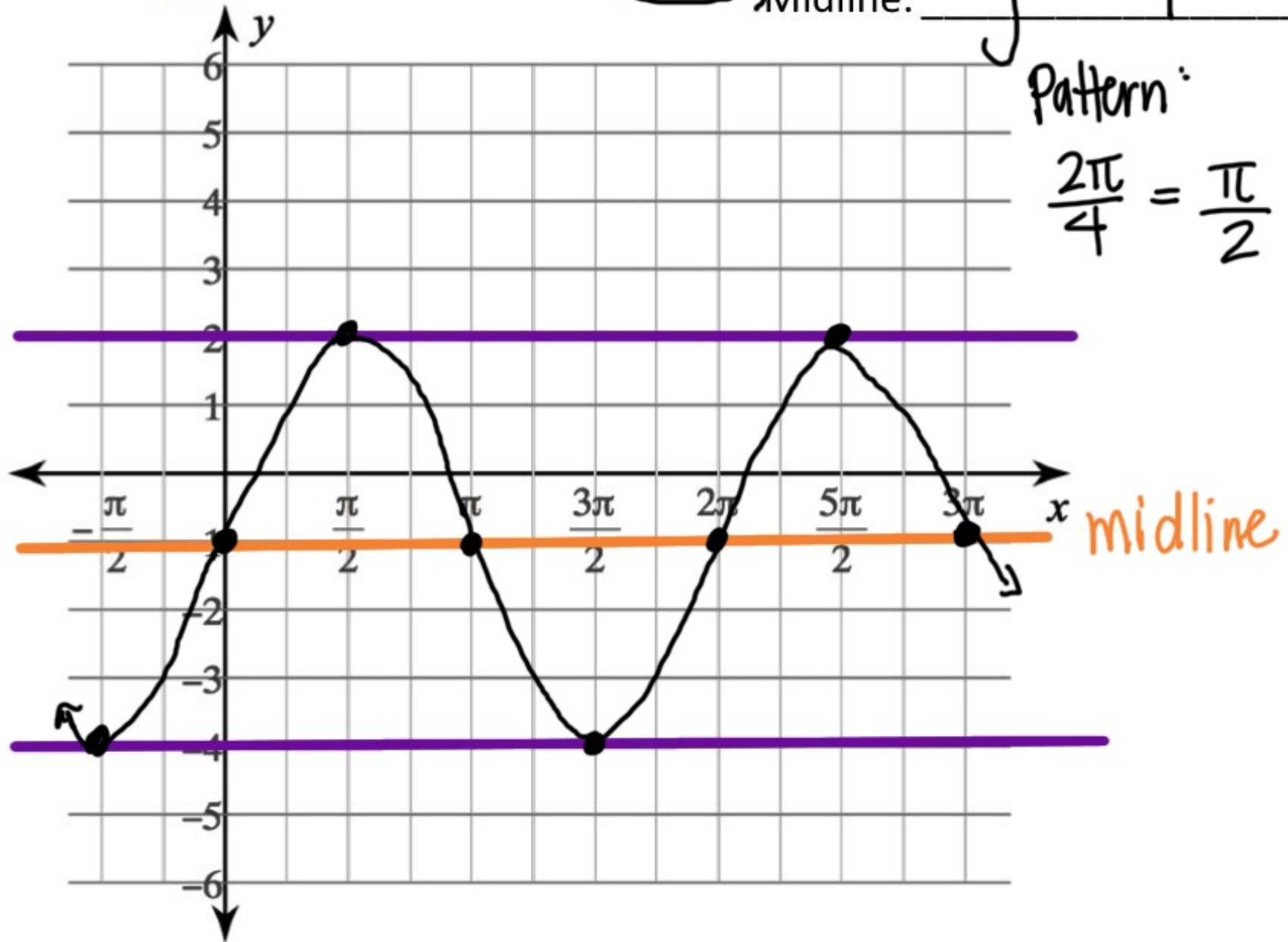
E. $y = 3 \sin \theta - 1$

Amplitude: 3

Midline: $y = -1$

Pattern:

$$\frac{2\pi}{4} = \frac{\pi}{2}$$



F. $y = -4 \overset{\text{flip}}{\sin} \theta + 1$

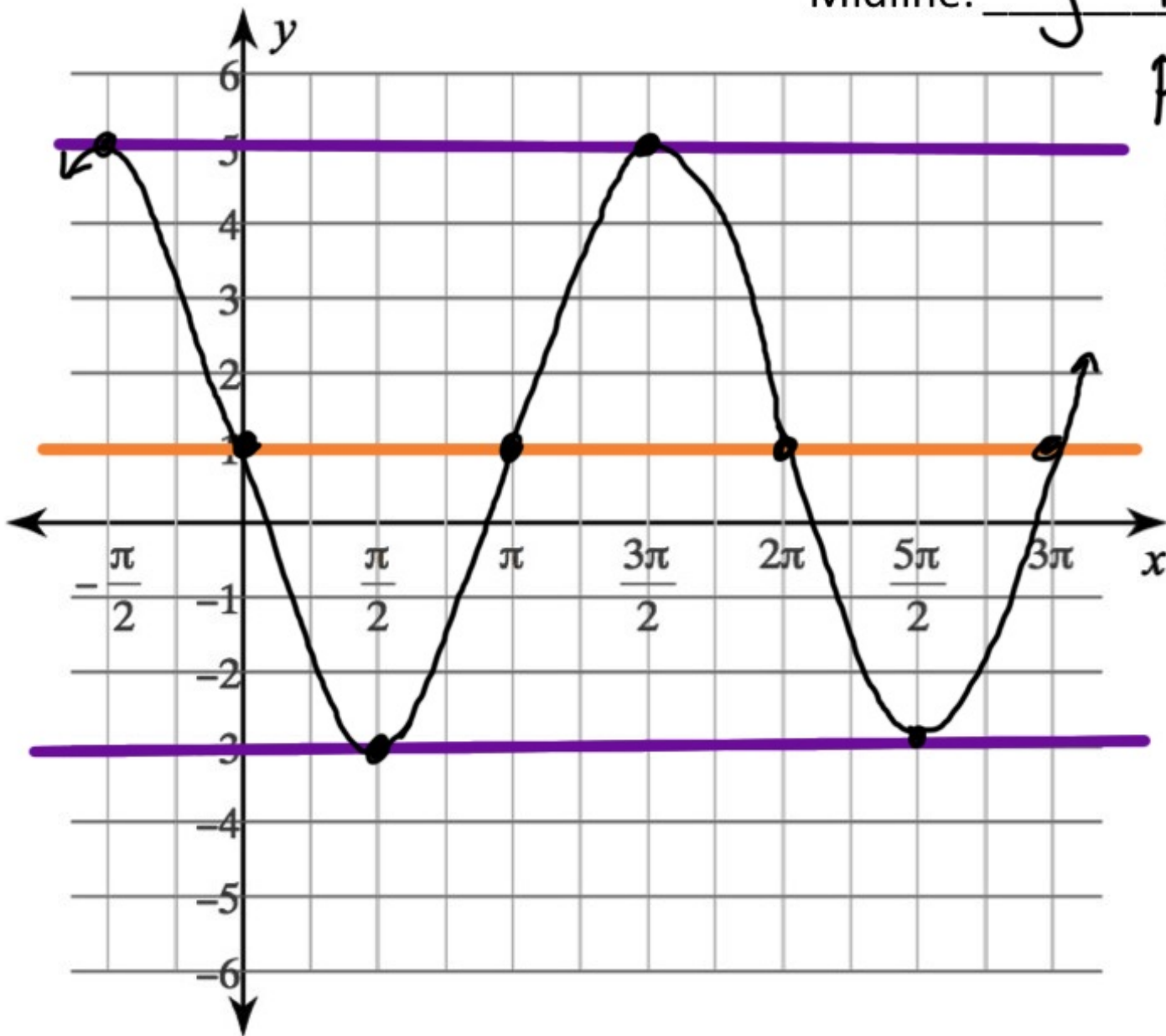
Amplitude: 4 (always +)

Midline: $y = 1$

period: 2π

pattern:

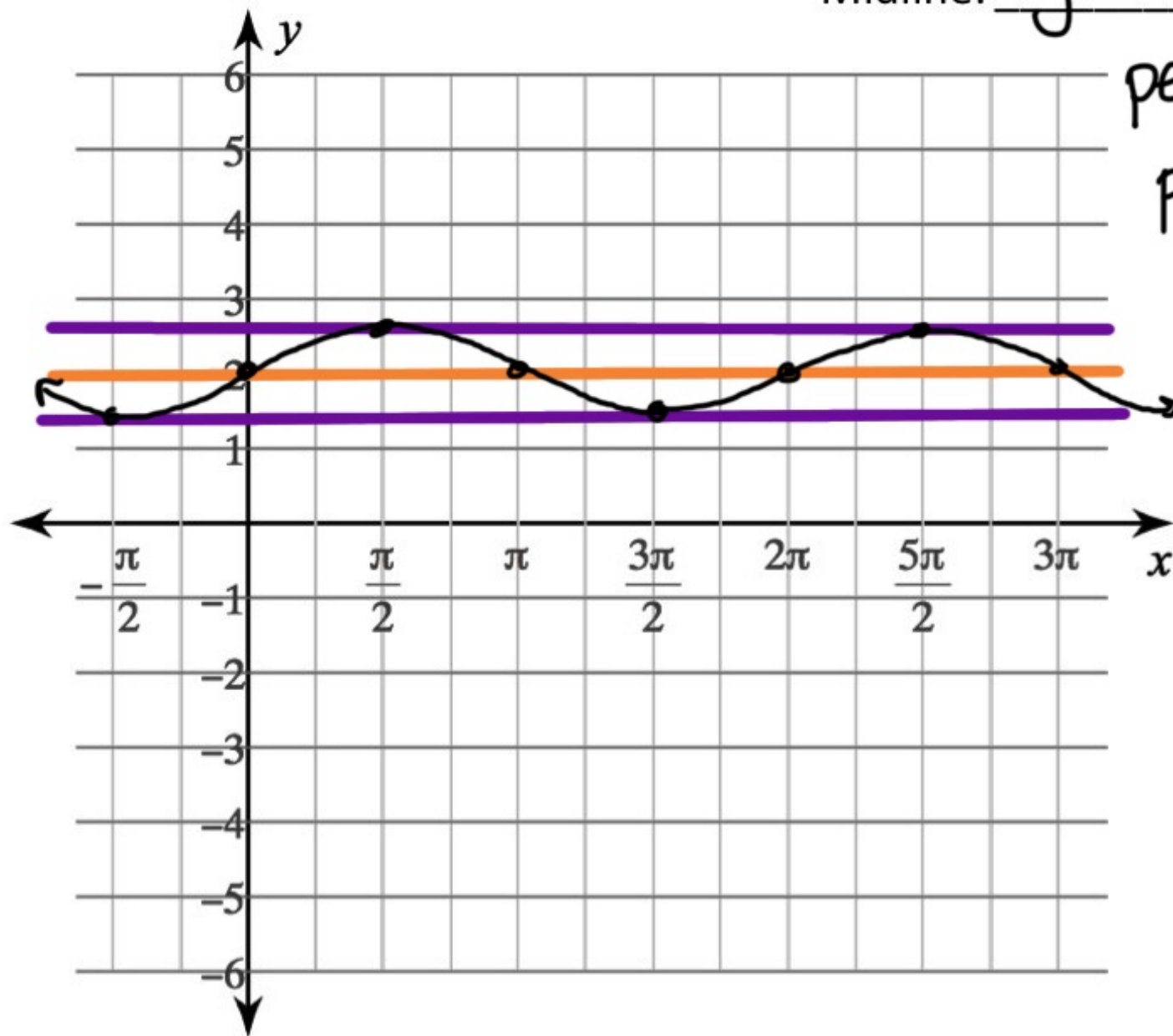
$$\frac{2\pi}{4} = \frac{\pi}{2}$$



G. $y = \frac{1}{2} \sin \theta + 2$

Amplitude: $\frac{1}{2}$

Midline: $y = 2$



period: 2π

pattern:

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

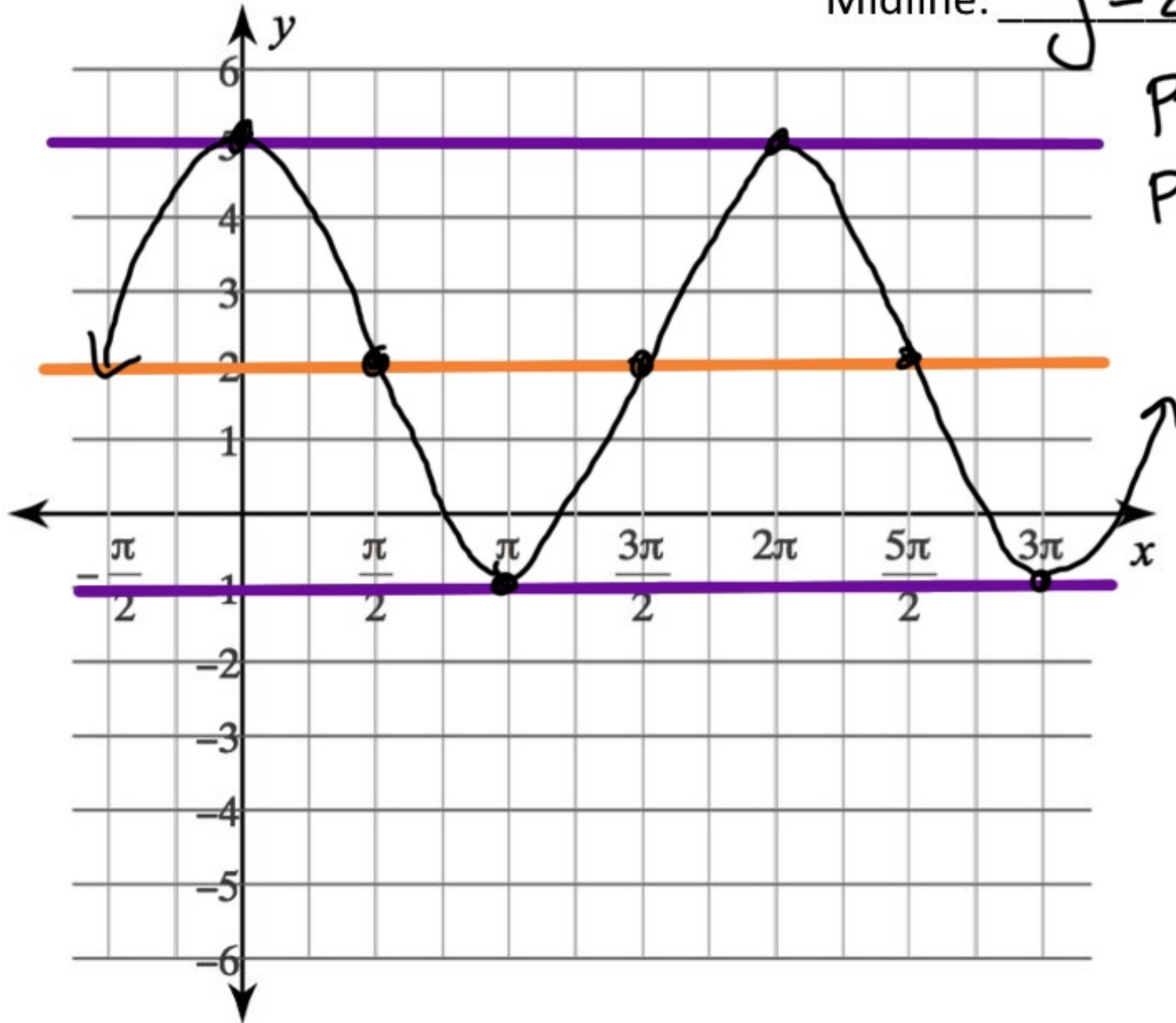
H. $y = 3 \cos \theta + 2$

Amplitude: 3

Midline: $y = 2$

period: 2π

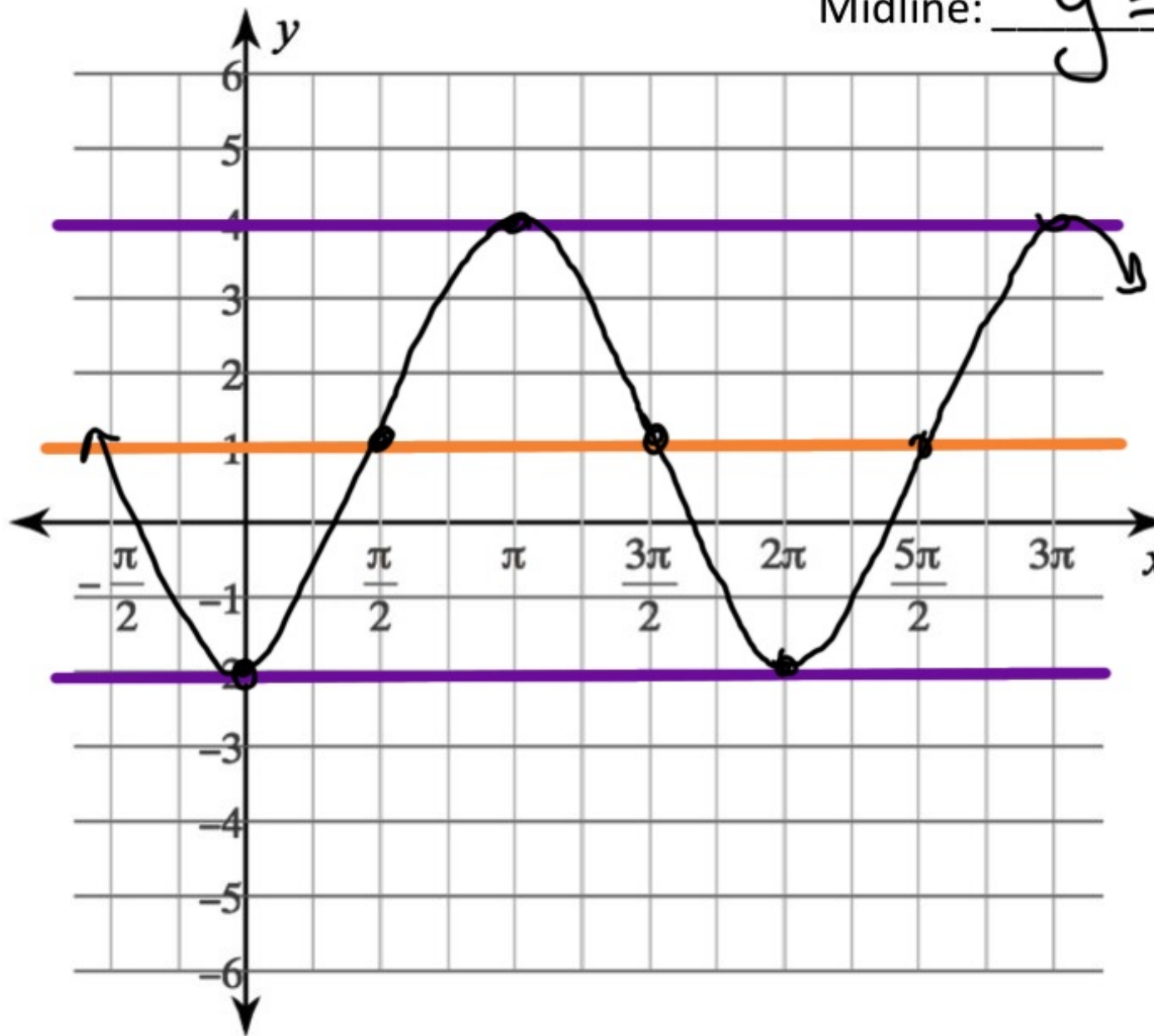
pattern: $\frac{2\pi}{4} = \frac{\pi}{2}$



1. $y = \overset{\text{flip}}{-3} \cos \theta + 1$

Amplitude: 3

Midline: $y = 1$



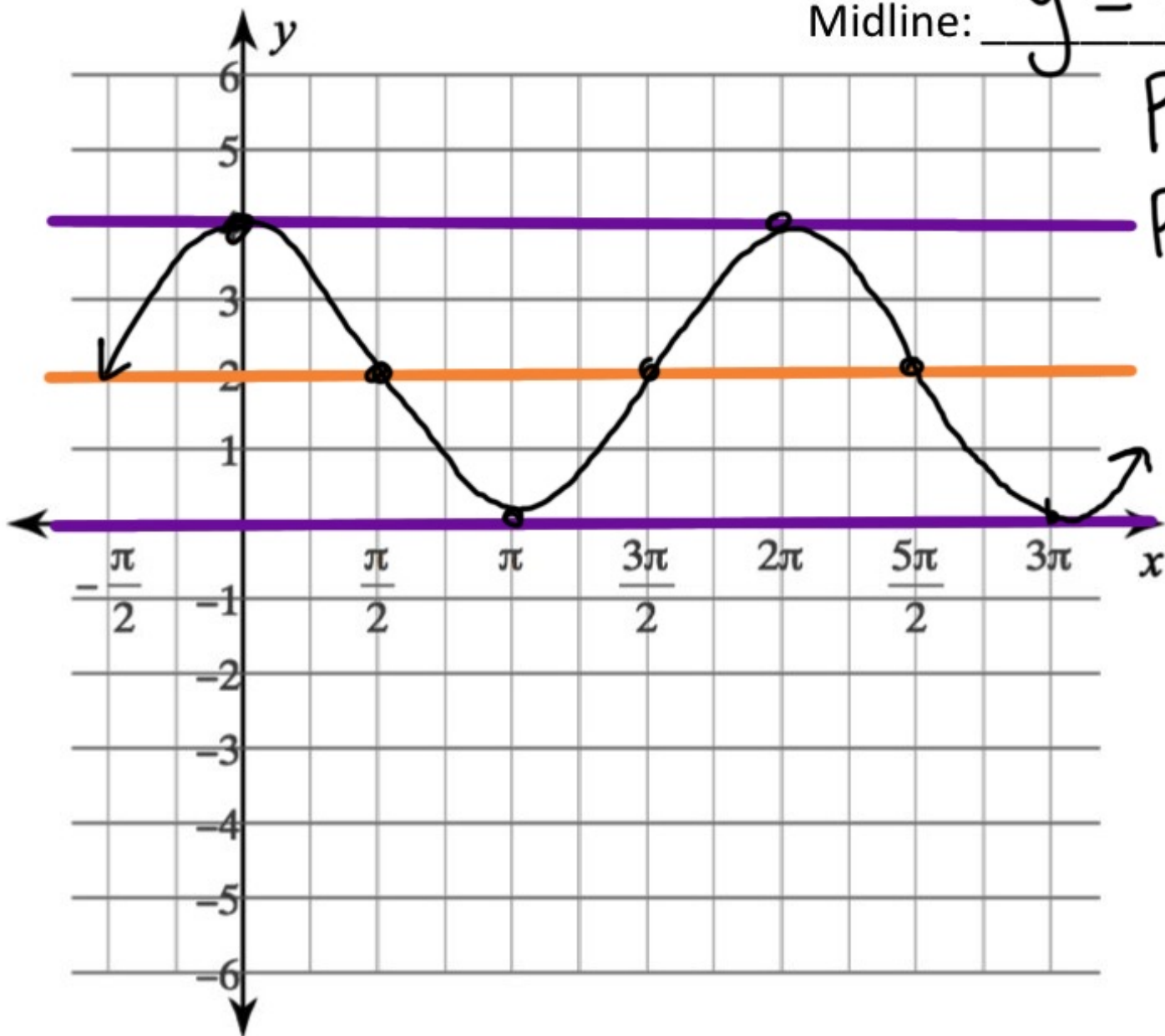
J. $y = 2 \cos \theta + 2$

Amplitude: 2

Midline: $y = 2$

PERIOD: 2π

PATTERN: $\frac{2\pi}{4} = \frac{\pi}{2}$



A. $y = 2 \sin\left(3\left(\theta - \frac{\pi}{4}\right)\right) + 1$

Amplitude: 2

Frequency: 3

Period: $\frac{2\pi}{3}$

Phase shift: right $\frac{\pi}{4}$
(left/right)

Midline: $y = 1$

B. $y = -1 \cos\left(\theta + \frac{\pi}{6}\right) - 4$

Amplitude: 1

Frequency: 1

Period: $\frac{2\pi}{1} = 2\pi$

Phase shift: left $\frac{\pi}{6}$

Midline: $y = -4$

C. $y = -3 \cos\left(\frac{1}{4}(\theta + \pi)\right) + 0$

Amplitude: 3

Frequency: $\frac{1}{4}$

Period: $\frac{2\pi}{\frac{1}{4}} = \frac{2\pi}{1} \cdot \frac{4}{1} = 8\pi$

Phase shift: left π

Midline: $y = 0$

D. $y = 2 \cos\left(4\left(\theta - \frac{\pi}{2}\right)\right) - 2$

Amplitude: 2

Frequency: 4

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Phase shift: Right $\frac{\pi}{2}$

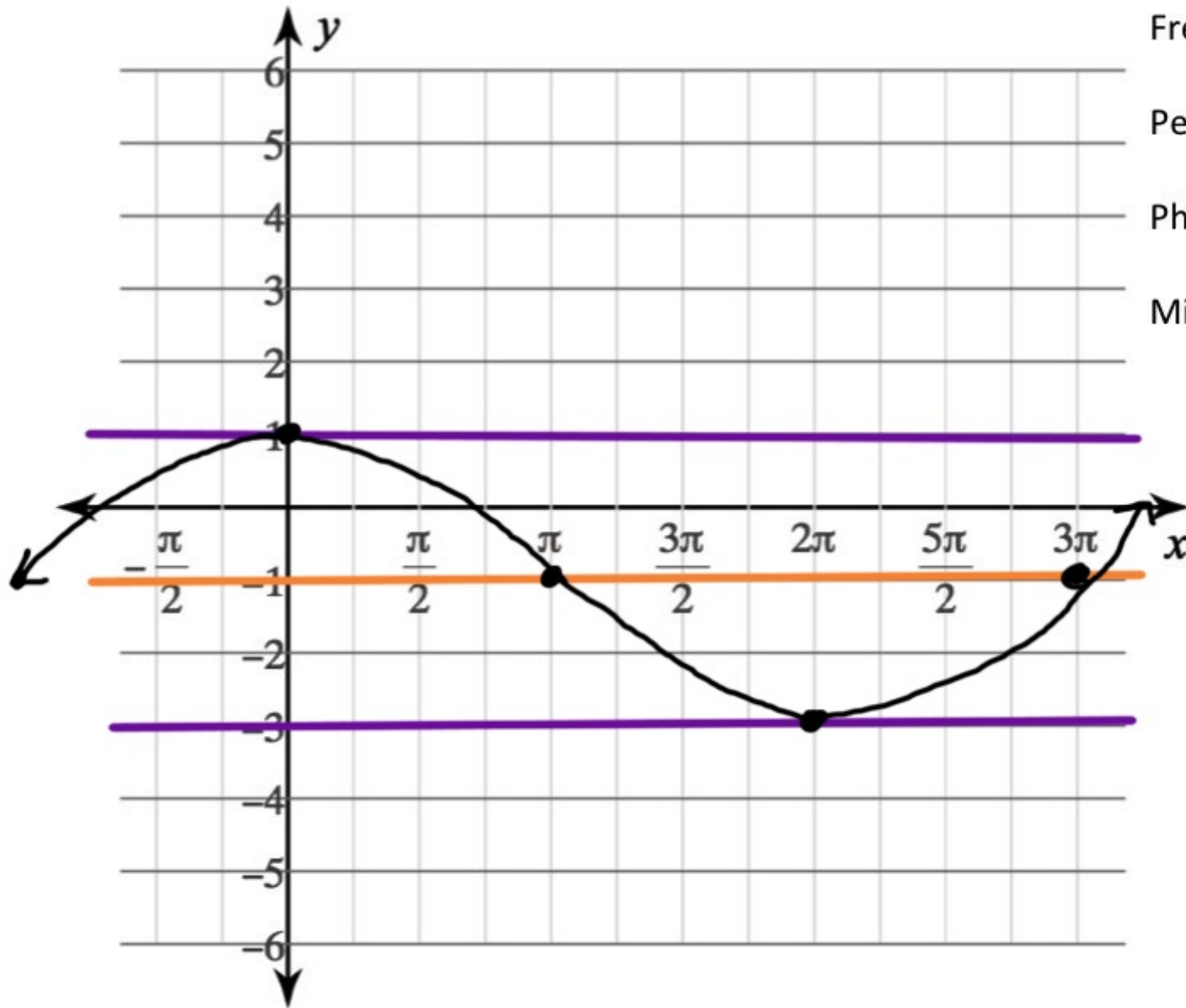
Midline: $y = -2$

Steps to graphing:

1. Identify the parent function. *(sine/cosine)*
2. Determine the amplitude, midline, period, and phase shift.
3. Mark the midline and phase shift on the graph (this is your new x- and y-axis).
4. Mark the 5-point pattern on the graph ($\frac{\text{period}}{4}$).
5. Sketch pretty WAVE.

(on pg. 16 of the notes)

$$E. y = 2 \cos\left(\frac{1}{2}\theta\right) - 1$$



Amplitude: 2

Frequency: $\frac{1}{2}$

Period: $\frac{2\pi}{1/2} = \frac{2\pi}{1} \cdot \frac{2}{1} = 4\pi$

Phase shift: None

Midline: $y = -1$

pattern:

$$\frac{4\pi}{4} = \pi$$

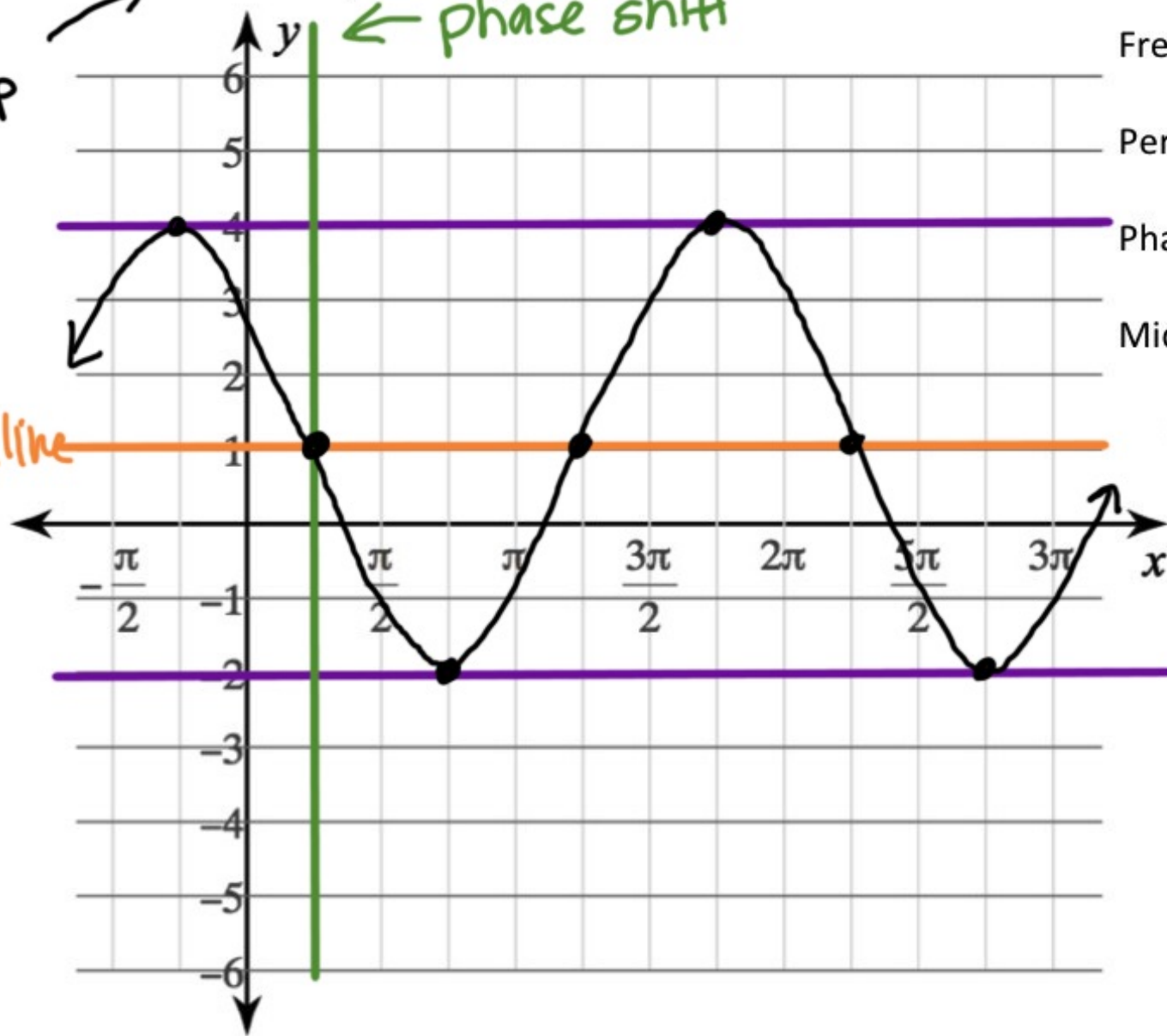
period

F. $y = -3 \sin\left(\theta - \frac{\pi}{4}\right) + 1$

flip

midline

← phase shift



Amplitude: 3

Frequency: 1

Period: $\frac{2\pi}{1} = 2\pi$

Phase shift: Right $\frac{\pi}{4}$

Midline: $y=1$

pattern: $\frac{2\pi}{4} = \frac{\pi}{2}$

period

G. $y = \frac{1}{2} \sin \left(\theta + \frac{\pi}{2} \right) + 2$

Amplitude: $\frac{1}{2}$

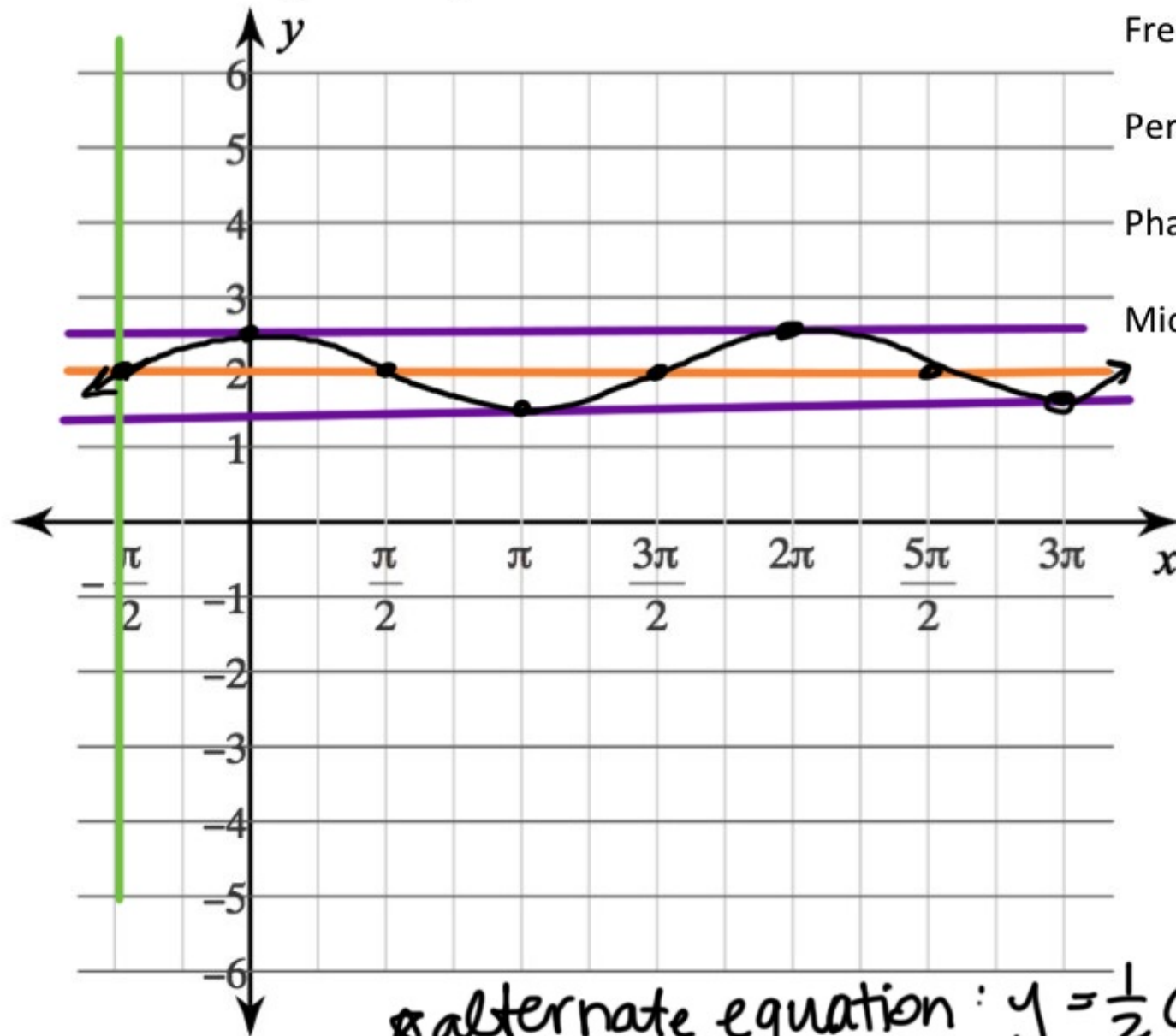
Frequency: 1

Period: $\frac{2\pi}{1} = 2\pi$

Phase shift: left $\frac{\pi}{2}$

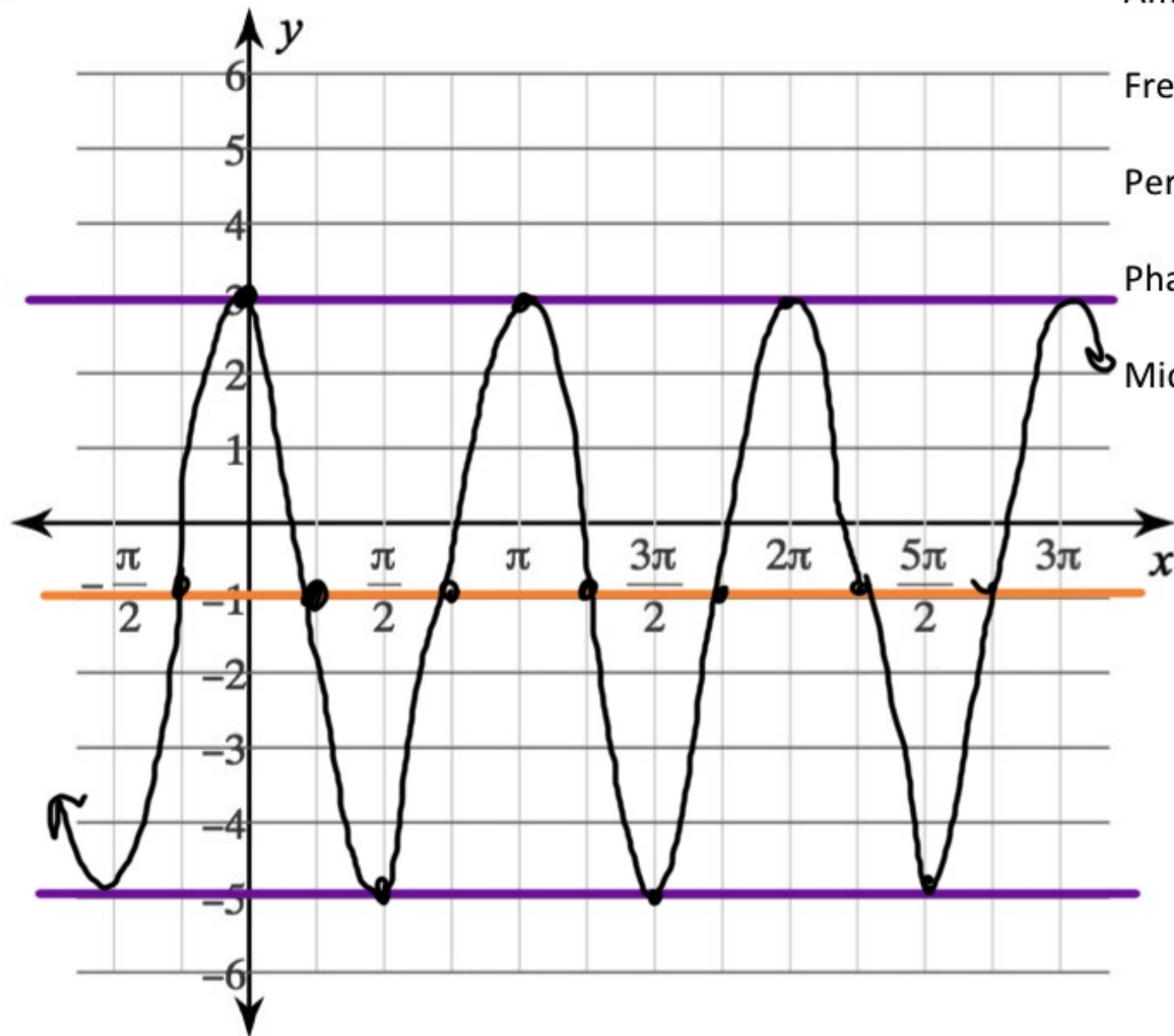
Midline: $y = 2$

pattern:
 $\frac{2\pi}{4} = \frac{\pi}{2}$



* alternate equation: $y = \frac{1}{2} \cos \theta + 2$

H. $y = 4 \cos(2\theta) - 1$



Amplitude: 4

Frequency: 2

Period: $\frac{2\pi}{2} = \pi$

Phase shift: None

Midline: $y = -1$

pattern:

$\frac{\pi}{4}$

period

- I. The cosine function that has been translated 2 units down and 5 units to the left

midline: $y = -2$ phase shift

$$y = \cos(\theta + 5) - 2$$

- J. A sine function with an amplitude of 3 and a period of $\frac{\pi}{2}$ that has been shifted down 2 units and right 3 units

$$a = 3$$

$$\text{period: } \frac{\pi}{2}$$

$$\text{midline: } y = -2$$

$$\text{phase shift: right 3}$$

$$y = 3 \sin\left(\frac{4}{1}(\theta - 3)\right) - 2$$

$$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi/2}$$

$$= \frac{2\pi}{1} \cdot \frac{2}{\pi} = 4$$