9.2 Angles and the Unit Circle

Standard Position: An angle is in standard position when the vertex is at the point (0,0)

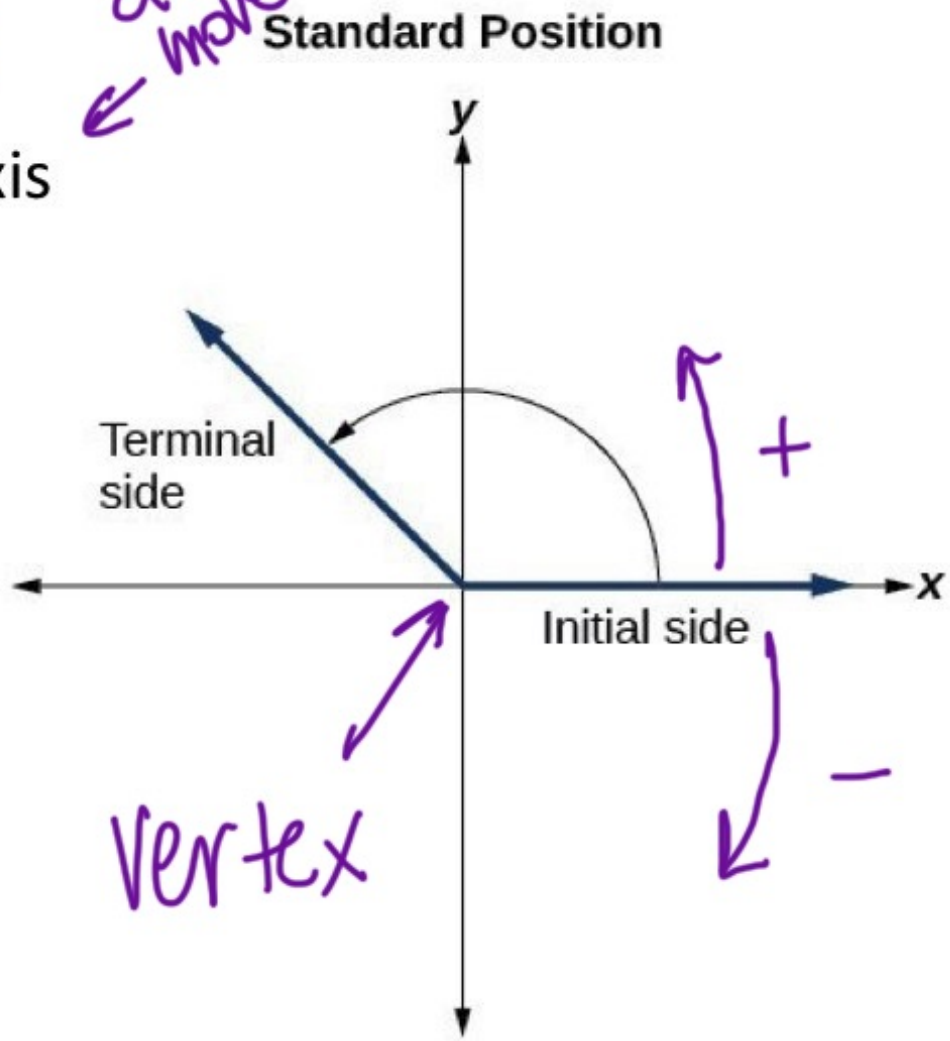
doesn't move

Initial Side: The ray on the positive X-axis

Terminal Side: a ray that rotates from the initial side

Positive Angles rotate counter-clockwise (opens up)

Negative Angles rotate clockwise (opens down)

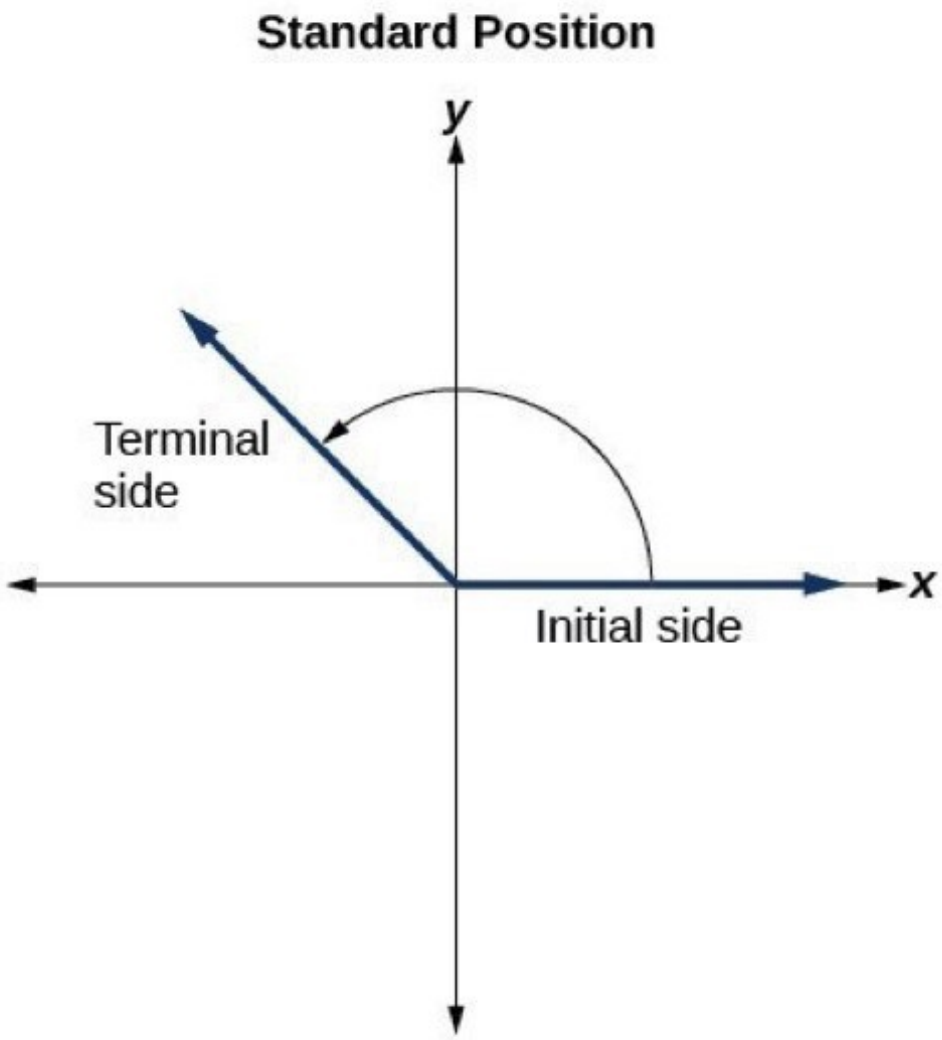


How many degrees make up a full rotation?

360°

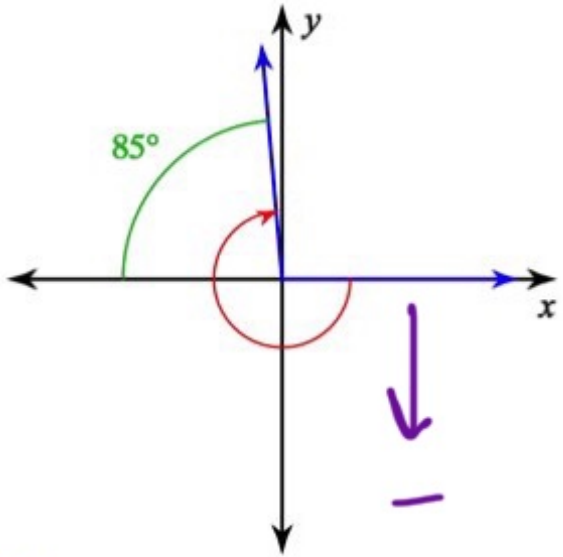
How many degrees make up a half rotation?

180°



EXAMPLE **Determine the measure of each angle.**

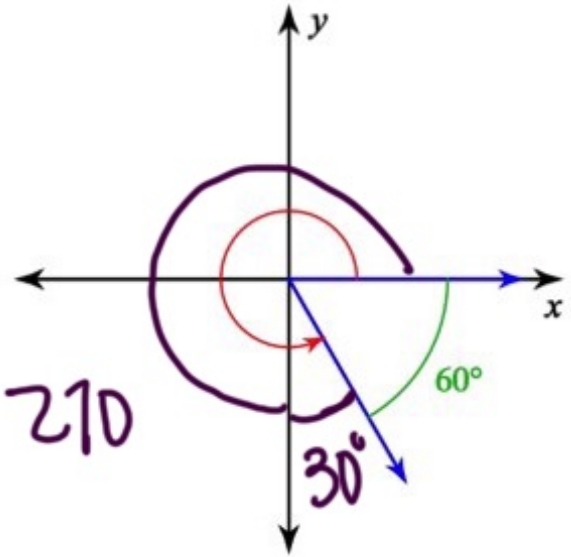
A.



$180 + 85 = 265$

$-265°$

B.



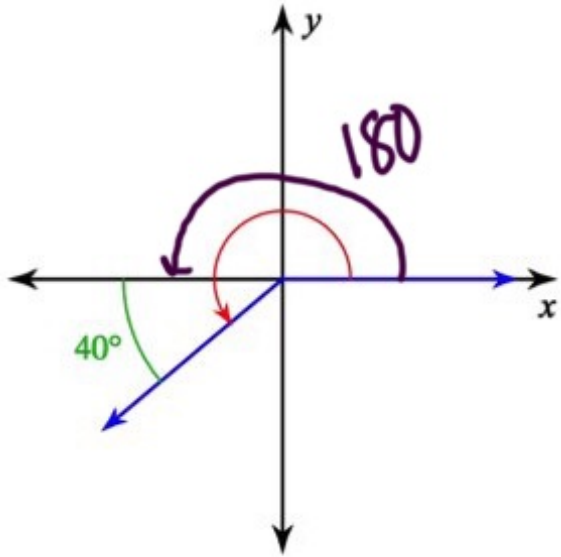
$270 + 30 = 300$

OR

$360 - 60 = 300$

$300°$

C.



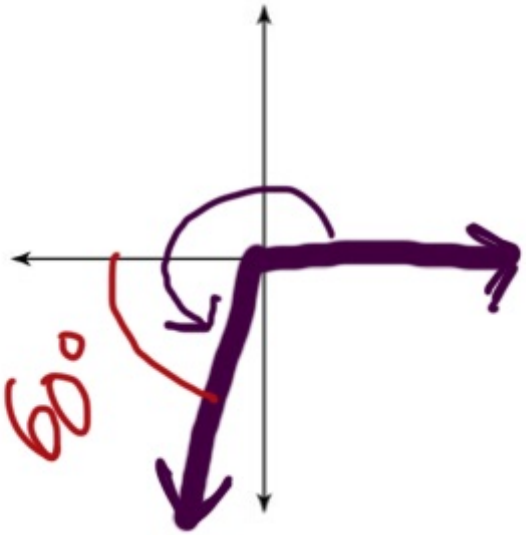
$180 + 40 = 220$

$220°$

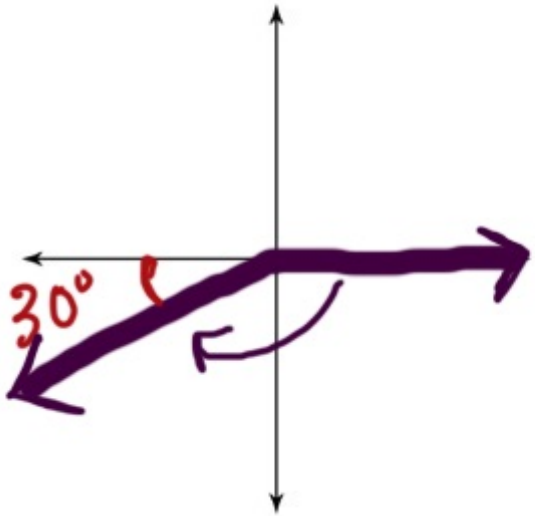
EXAMPLE

Draw each angle in standard position. What quadrant is the terminal side in?

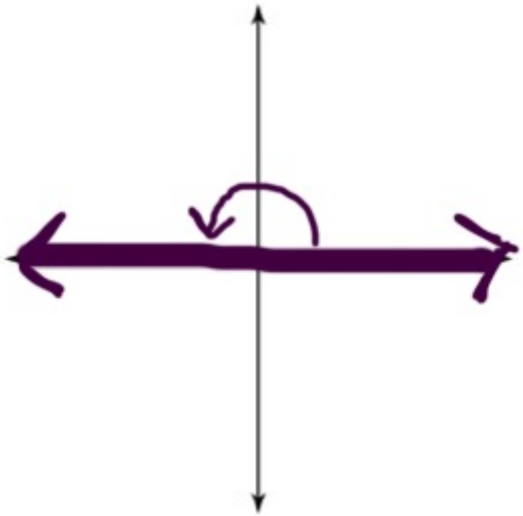
D. $240°$



E. $-150°$

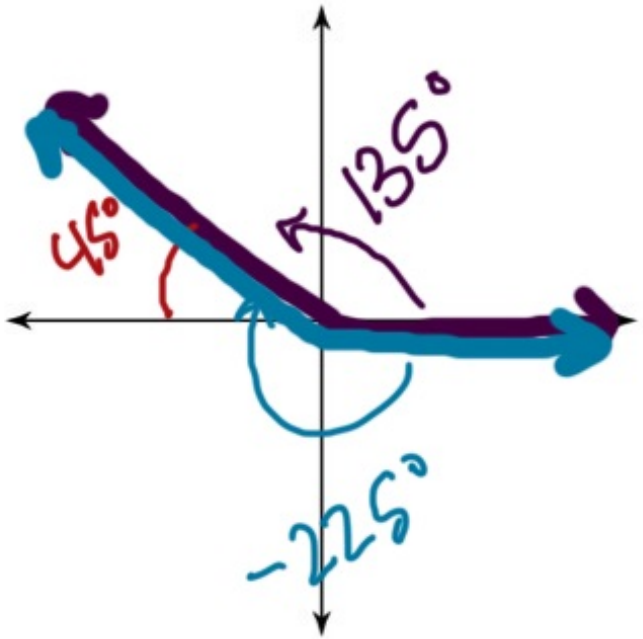


F. $180°$



EXAMPLE

G. Draw 135° and –225° on the same coordinate plane. What do you notice?

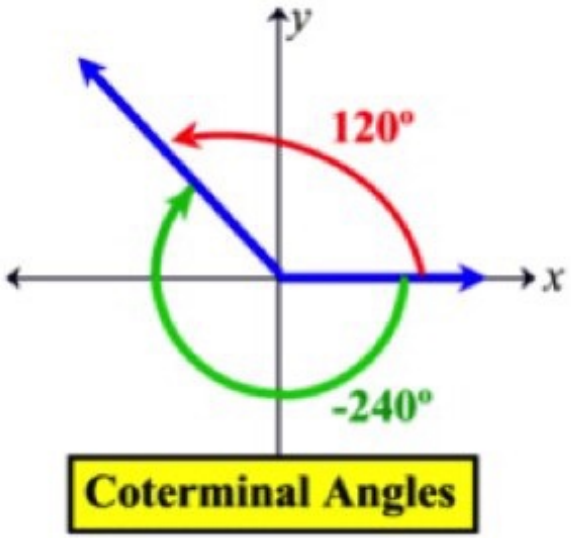


same angle

Coterminal angle are angles in standard position with the same terminal side.

Strategy for finding coterminal angles	$\pm 360°$

(as many times as you want)



EXAMPLE

List three coterminal angles for each of the following. Make sure at least one of them is negative.

H. $60°$

$60 - 360 = -300°$

$60 + 360 = 420°$

$420 + 360 = 780°$

I. $500°$

$140°$

$-220°$

$-580°$

$860°$

$1220°$

$1580°$

etc.

J. $-25°$

$335°$

$695°$

$1055°$

$-385°$

$-745°$

$-1105°$

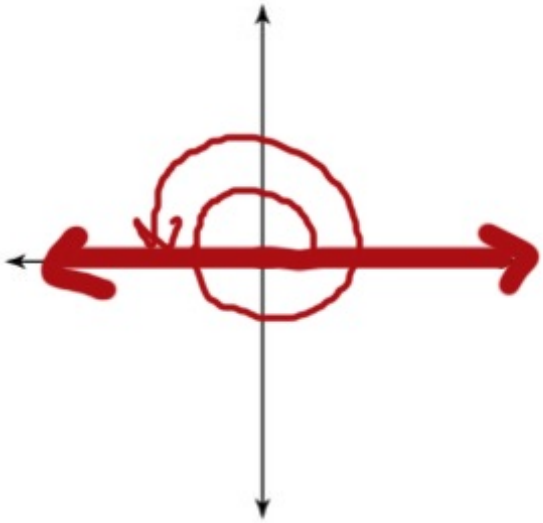
etc.

EXAMPLE

Draw each angle on the coordinate plane.

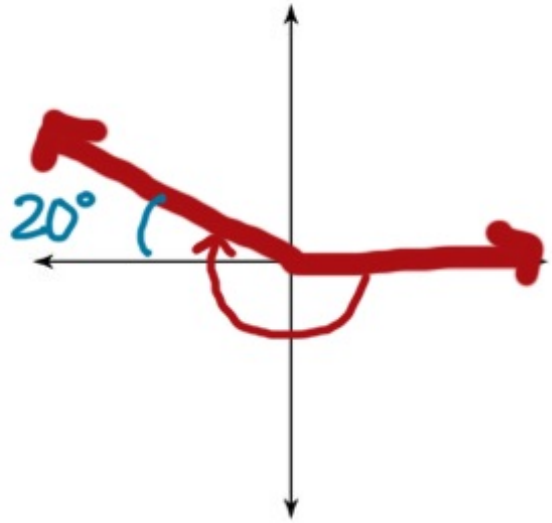
★ Find a coterminal angle between 0 & 360

K. $540°$



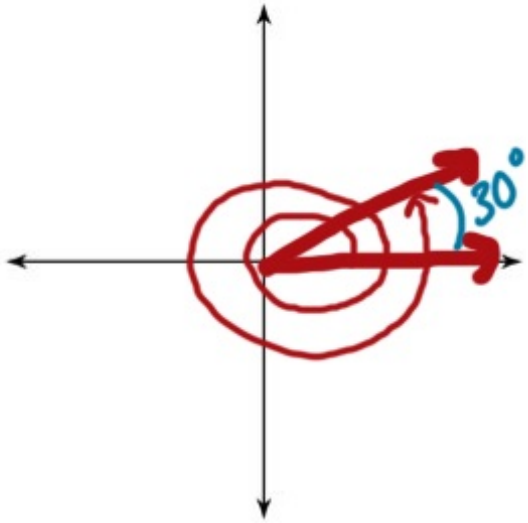
$540 - 360 = 180°$

L. $-200°$



$-200 + 360 = 160°$

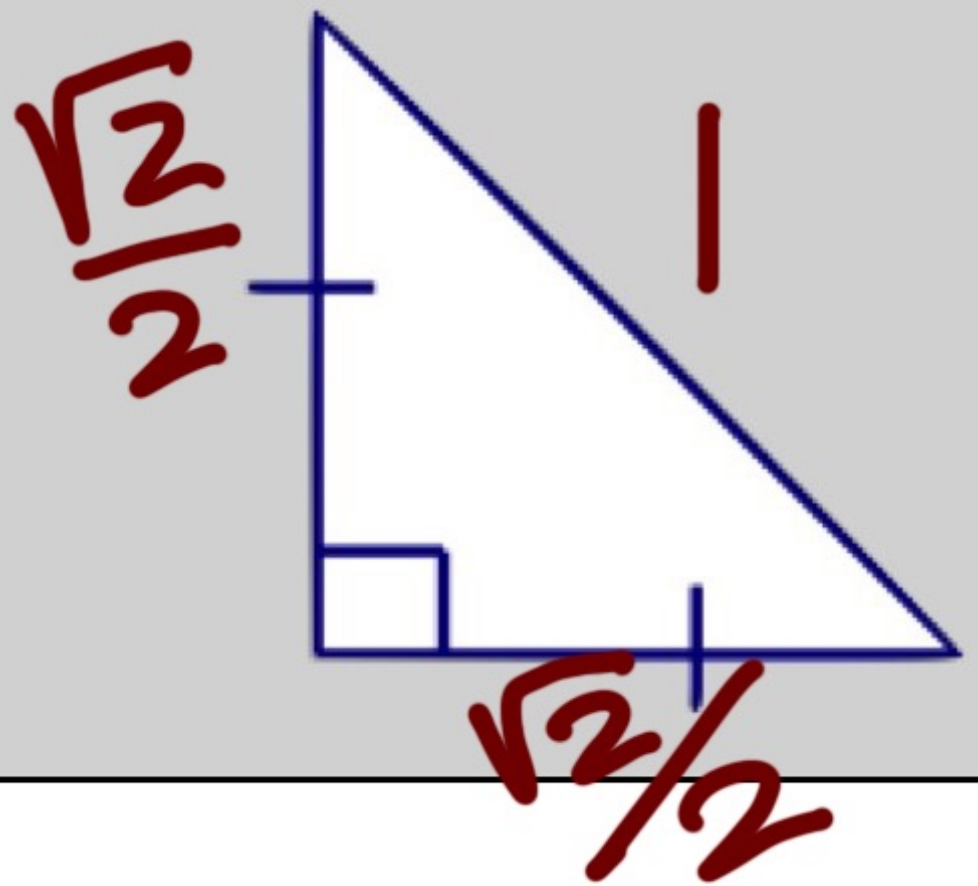
M. $750°$



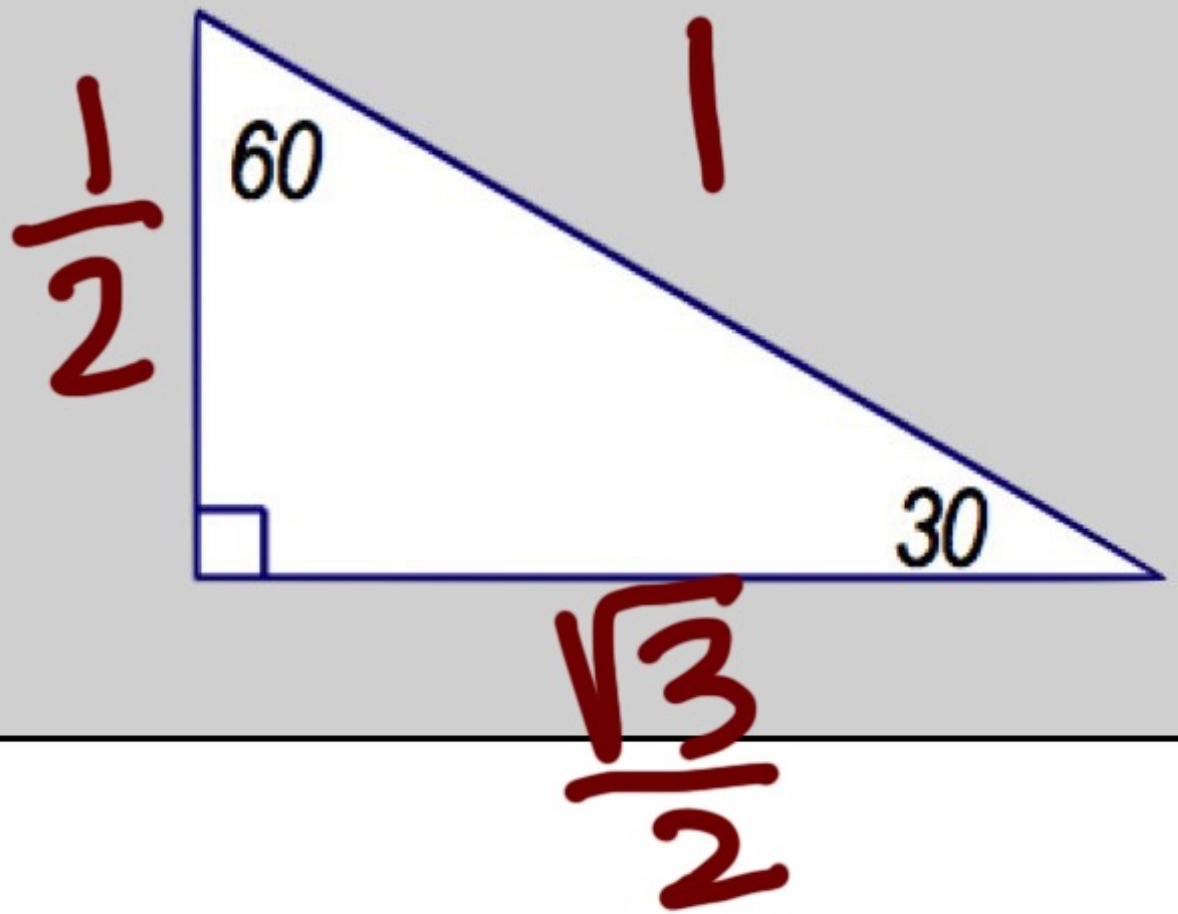
$750 - 360 = 390$

$390 - 360 = 30$

45°-45°-90° Triangle



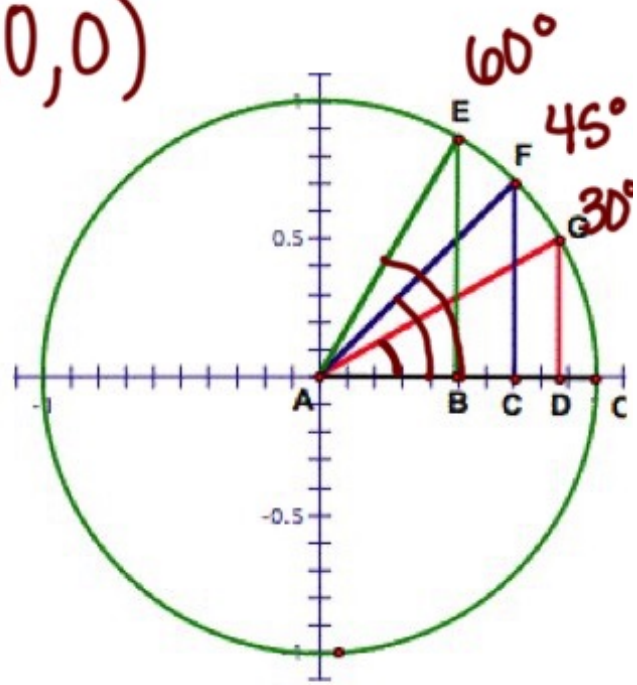
30°-60°-90° Triangle



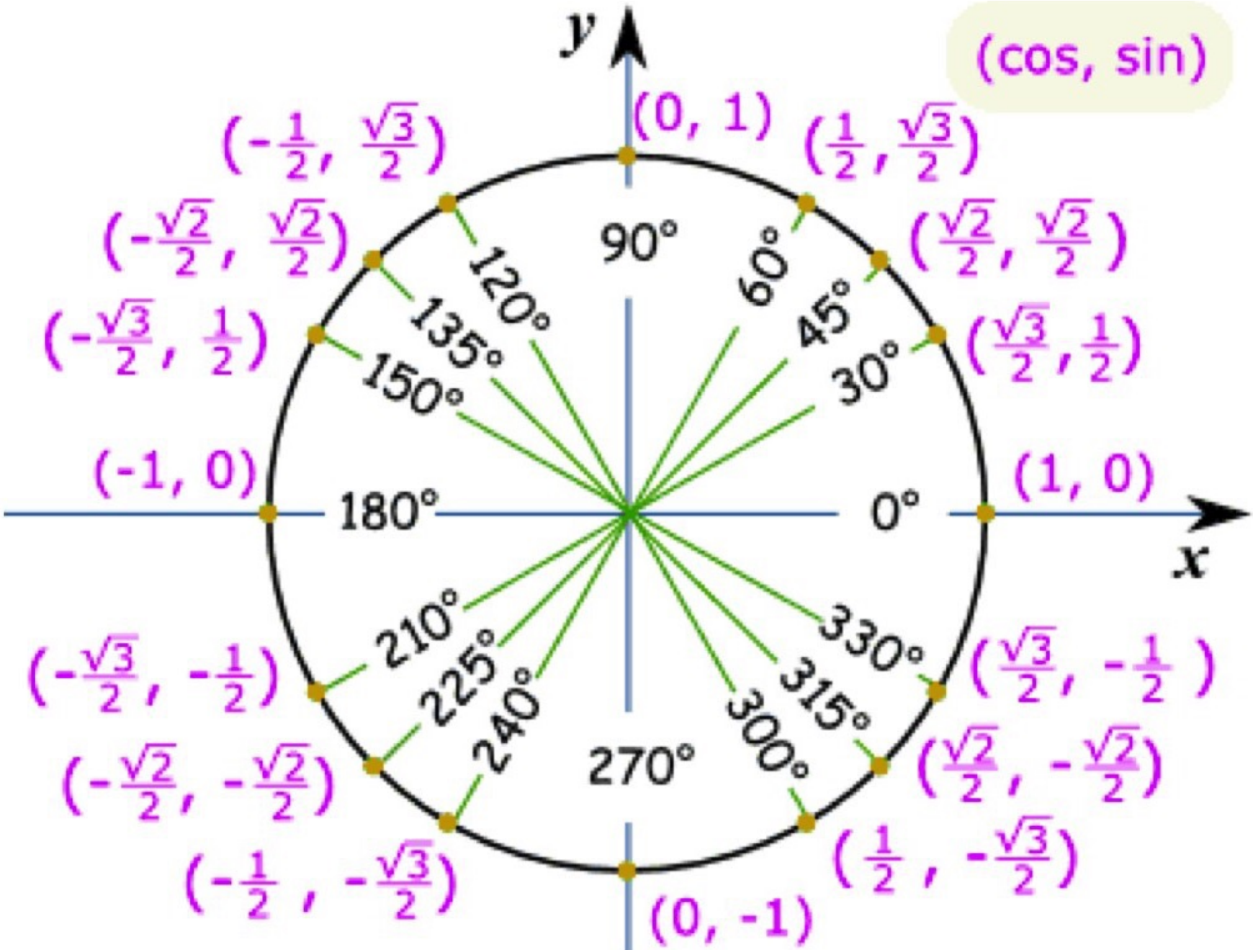
The Unit Circle

The Unit Circle is a circle with a radius of 1. It is segmented into 3 different triangles per quadrant, with interior angles of 30°, 45°, and 60°. The figure to the right illustrates this concept for the first quadrant.

center : (0,0)



We can use the relationships above to fill out the missing information for the unit circle below.


(cos, sin)
y
x
90°
60°
45°
30°
0°
120°
135°
150°
180°
210°
225°
240°
270°
300°
315°
330°
(0, 1)
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
(1, 0)
$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$
$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$
(0, -1)
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$
(-1, 0)
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\sin\theta =$ y	$\cos\theta =$ x	$\tan\theta =$ y ÷ x

EXAMPLE **Find the EXACT values of the following trig functions:**

N. $\sin 120°$ y $\frac{\sqrt{3}}{2}$

O. $\cos 270°$ x 0

P. $\tan 45°$ $y \div x$ $\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = 1$

Q. $\tan 330°$ $y \div x$ $\frac{1}{2} \div \frac{\sqrt{3}}{2}$

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

R. $\sin 135°$ y $\frac{\sqrt{2}}{2}$

S. $\cos 150°$ x $-\frac{\sqrt{3}}{2}$