9.4 Law of Sines and Cosines

Find each missing part of the triangle (sides and angles):
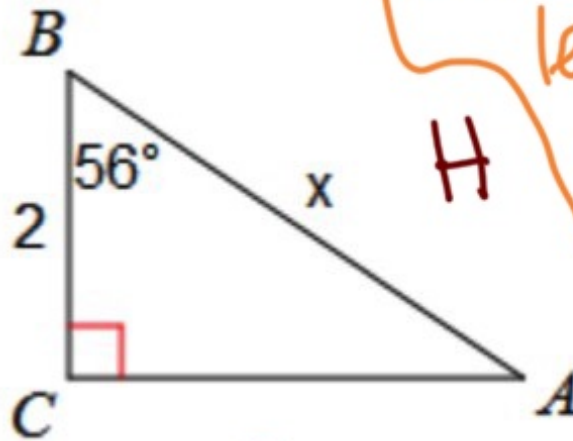
A.

B H A

θ

3   4

A   O

C
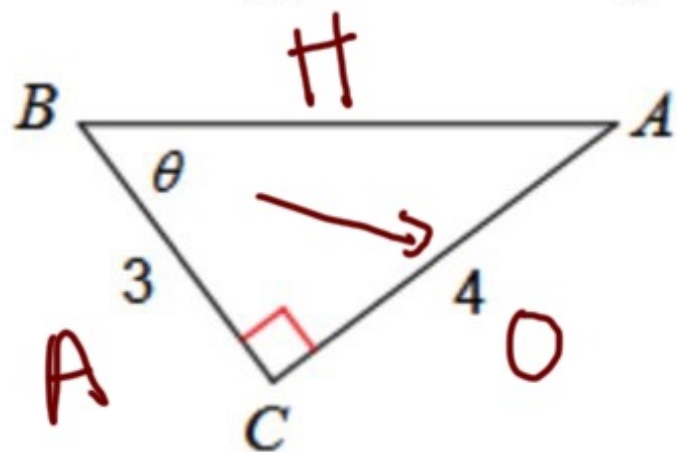
$$\cancel{tan}\cancel{(}\tan\theta)\overset{\tan^{-1}}{=}\left(\frac{4}{3}\right)$$

$$\boxed{\theta = 53°}$$

round angles to nearest degree

B.

* Round side lengths to hundredth

B

56°   X   H

2

A

C   A

O

$$X\cos 56° = \frac{2}{\cancel{X}}\cancel{X}$$

$$\frac{X\cos 56°}{\cancel{\cos 56°}} = \frac{2}{\cos 56°}$$

$$\boxed{X = 3.58}$$
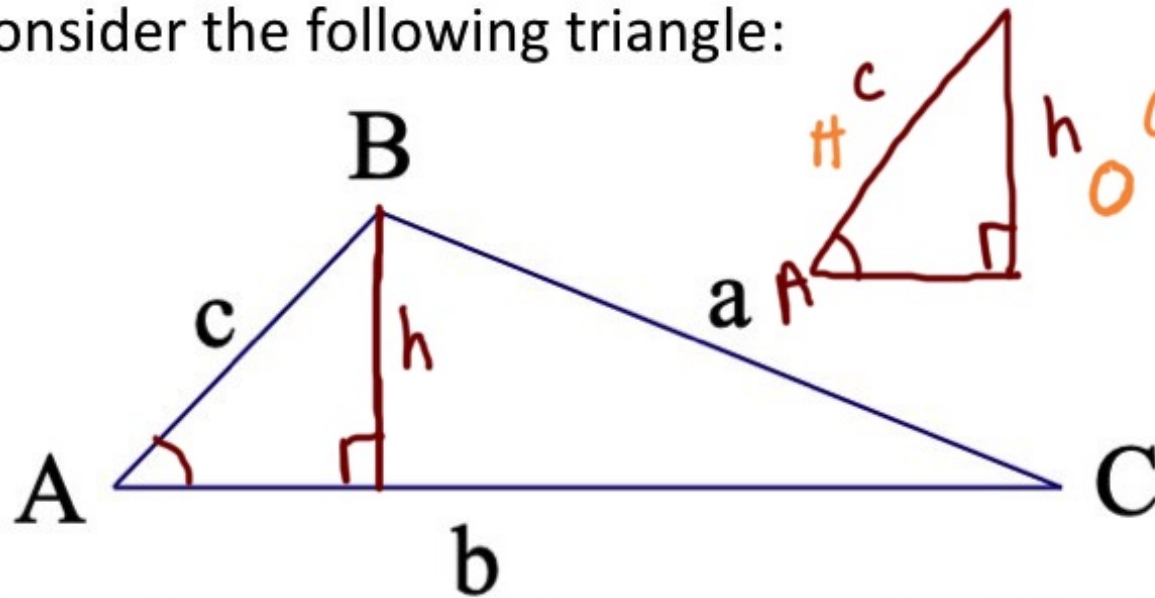
C. Find the area of the triangle:

3 ft

4 ft
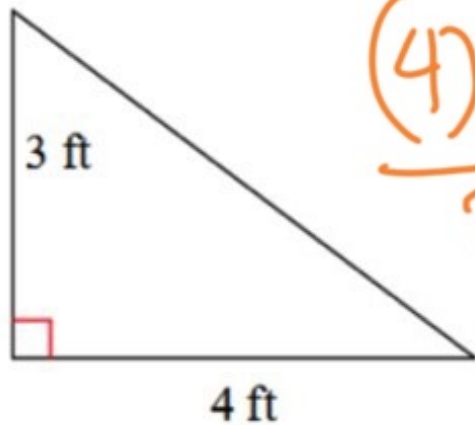
$$\frac{(4)(3)}{2} = 6 \text{ ft}^2$$

The area formula: $\dfrac{b \cdot h}{2}$

***only works when the height is
__perpendicular__ to the base

Consider the following triangle:

$$c \cdot \sin A = \frac{h}{c} \cdot c$$

$$h = c \cdot \sin A$$

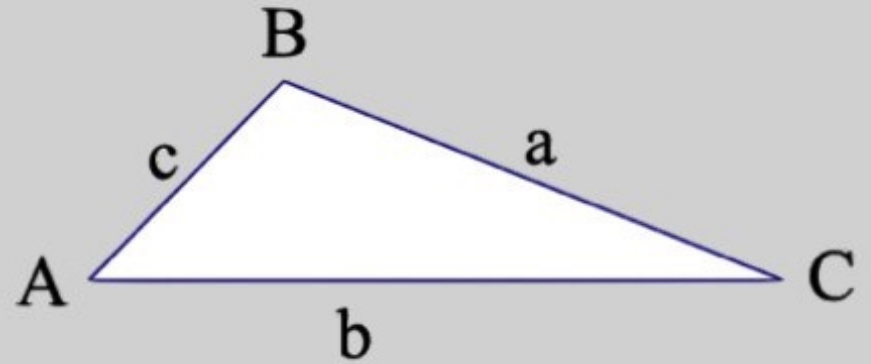$$\boxed{Area = \frac{b \cdot c \sin A}{2}}$$

If you know angle A, can you find the height of the triangle?

Area of Triangle:

$$\frac{1}{2} b \cdot c \cdot \sin A$$

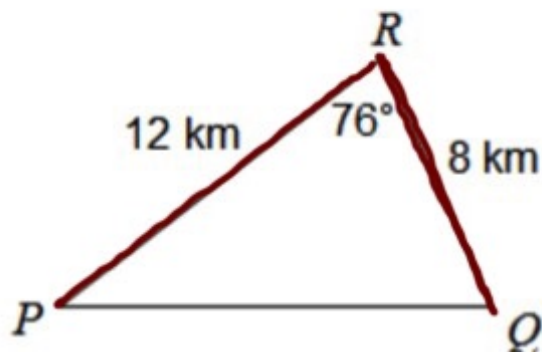Where *A* is the included angle of 2 consecutive sides, *b*, and *c*.

Area → use sine

* *b* & *c* are the sides that make ∠A

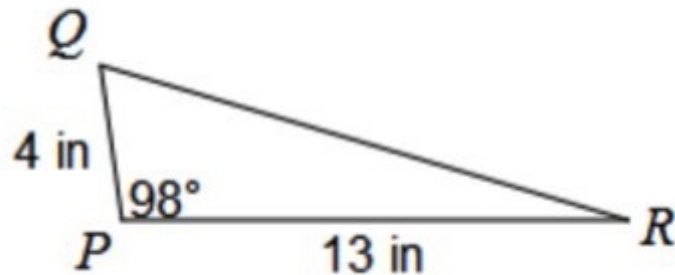**Find the area of each triangle. State if there is not enough information given.**

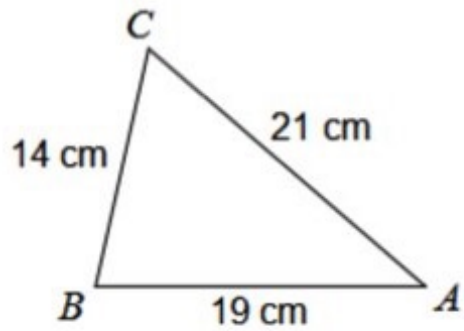D.

$$\frac{1}{2}(12)(8)\sin 76°$$

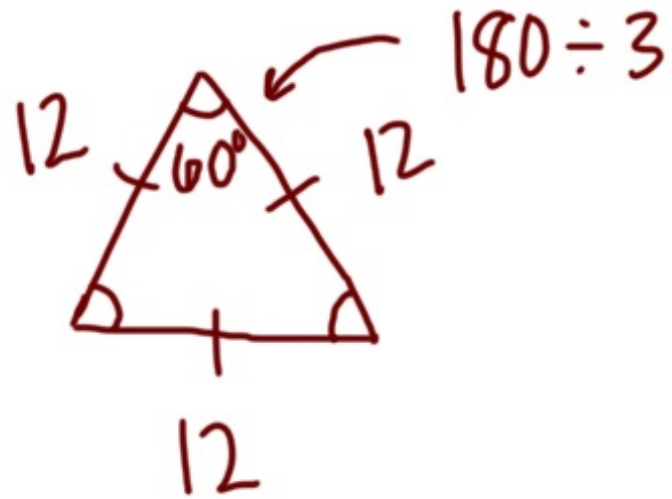$$46.57 \text{ km}^2$$

E.

$$\frac{1}{2}(4)(13)\sin 98°$$

$$25.75 \text{ in}^2$$

F.

C, 14 cm, 21 cm, B, 19 cm, A

Not enough info

G. Jaron is building a triangular garden in his yard. Each side of the triangle is the same. If the amount of fence he uses on the garden is 36 feet, find the area of the garden he is building.
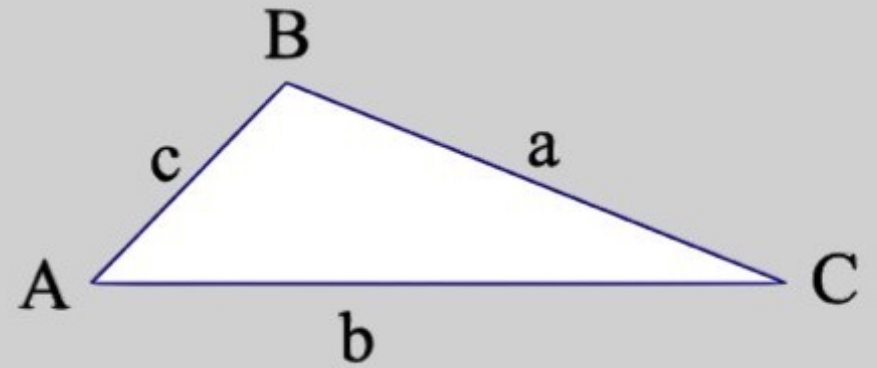

$180 \div 3$

12   60°   12

12

Area: $\frac{1}{2}(12)(12)\sin 60°$

$\boxed{62.35 \text{ ft}^2}$

LAW OF SINES

Find a side

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Find an angle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
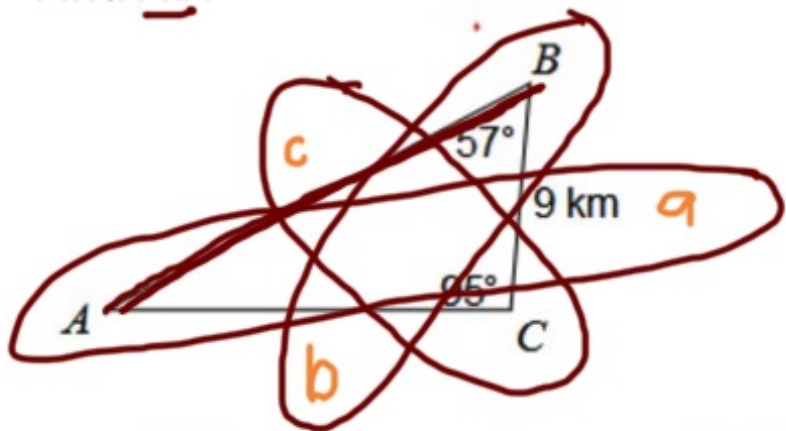
***The Law of Sines **ONLY** works when:

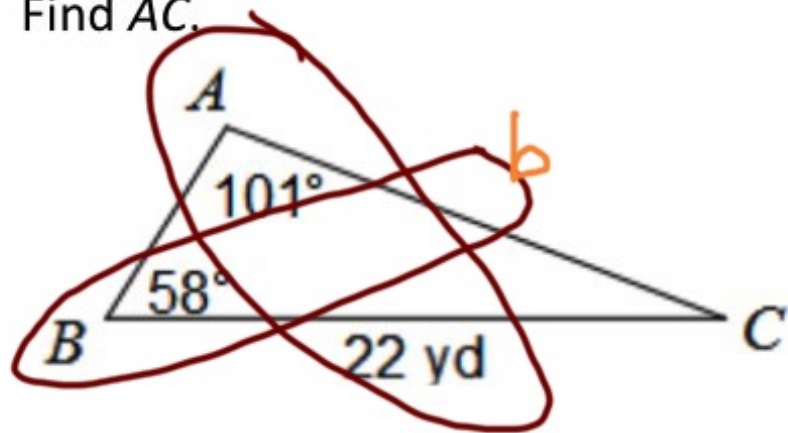you know angle & its opposite side

AAS

ASA

SSA

H. Find AB.

$$\boxed{\frac{c}{\sin 95°}} = \frac{\cancel{b}}{\cancel{\sin 57°}} = \boxed{\frac{9}{\sin A}}$$

$$A = 180 - 95 - 57 = 28°$$

$$\cancel{\sin 95°} \frac{c}{\sin 95°} = \frac{9}{\sin(28°)} \cdot \frac{\sin(95°)}{1}$$

$$\boxed{c = 19.10 \text{ km}}$$

I. Find AC.

$$\cancel{\sin 58°} \frac{b}{\sin 58°} = \frac{22}{\sin 101°} \cdot \frac{\sin 58°}{1}$$

$$\boxed{b = 19.01 \text{ yd}}$$

J. Find $m\angle B$.
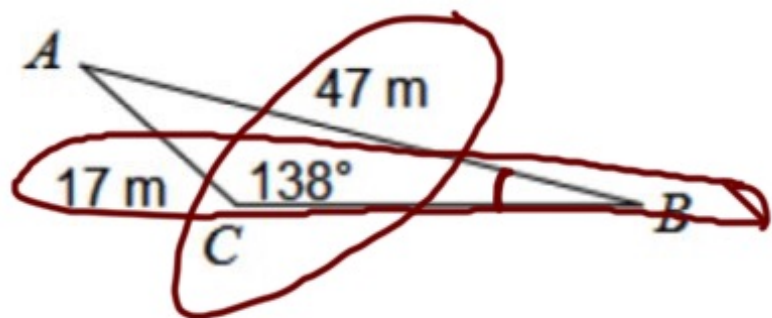
K. Find $m\angle B$.

$$17 \cdot \frac{\sin B}{17} = \frac{\sin(138°)}{47} \cdot \frac{17}{1}$$

$$\sin^{-1}(\sin B) \overset{\sin^{-1}}{=} (0.242)$$

$$\boxed{B = 14°}$$
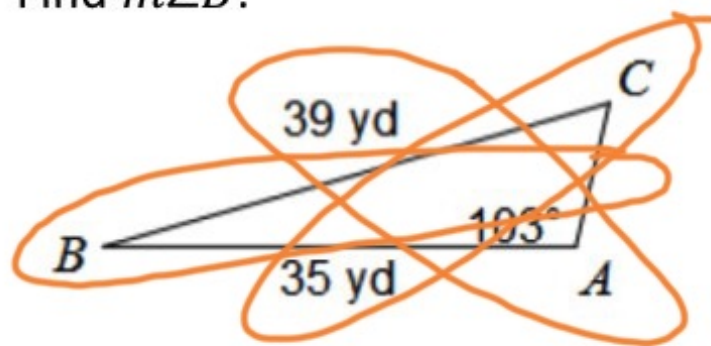
$$35 \cdot \frac{\sin C}{35} = \frac{\sin 103°}{39} \cdot 35$$

$$\sin^{-1}(\sin C) \overset{\sin^{-1}}{=} (0.874)$$

$$C = 61°$$

$$B = 180 - 61 - 103$$

$$\boxed{B = 16°}$$

L. Find AB.

13 km   C   114°   26°   A   B   c

$$\frac{\sin 114°}{1} \cdot \frac{13}{\sin 26°} = \frac{c}{\sin 114°} \cdot \frac{\sin 114°}{1}$$

$$c = 27.09 \text{ km}$$

M. Find $m\angle A$.

A   10 mi   c   28°   B   C   9 mi

$$\frac{9}{1} \cdot \frac{\sin A}{9} = \frac{\sin 28°}{10} \cdot \frac{9}{1}$$
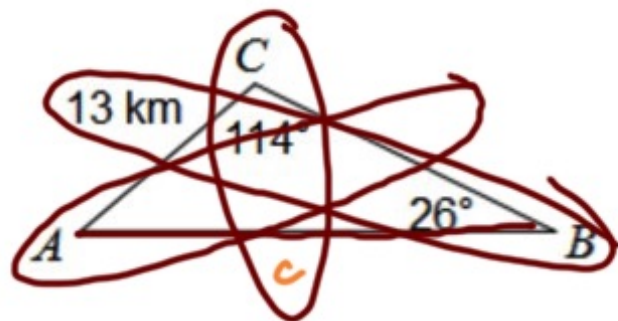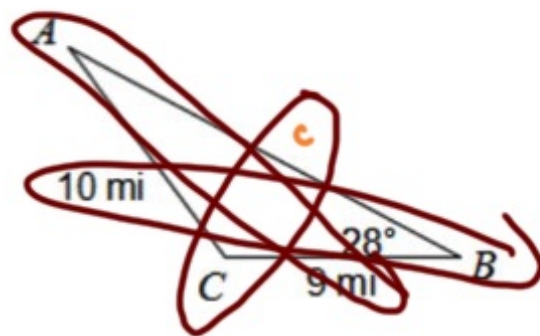
$$\sin^{-1}(\sin A) = \overset{\sin^{-1}}{(0.422)}$$

$$A = 25°$$

LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$
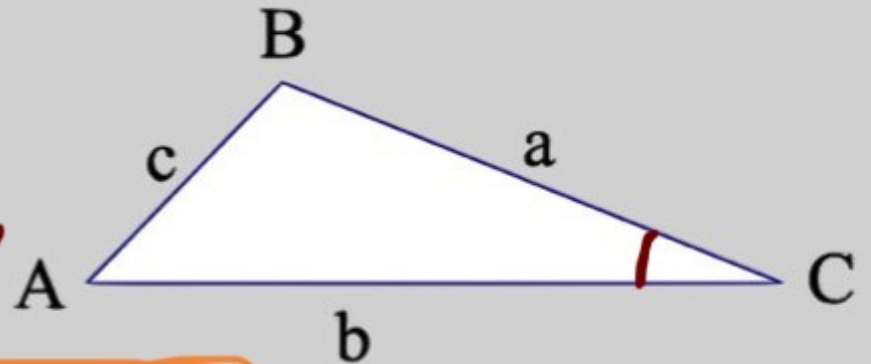
$(a \, \& \, b \text{ make up angle } C.)$

SAS   OR   SSS

N. Find *AC*.

B

24 mi   30 mi

130°

A   C

b
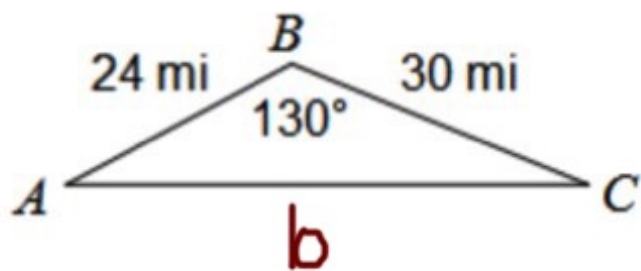
$$b^2 = 24^2 + 30^2 - 2(24)(30)\cos 130°$$

$$\sqrt{b^2} = \sqrt{2401.61}$$

$$\boxed{b = 49.01 \text{ mi}}$$

O. Find *AC*.

opposite sides of equation

C

b

28 m

127°

B   22 m   A
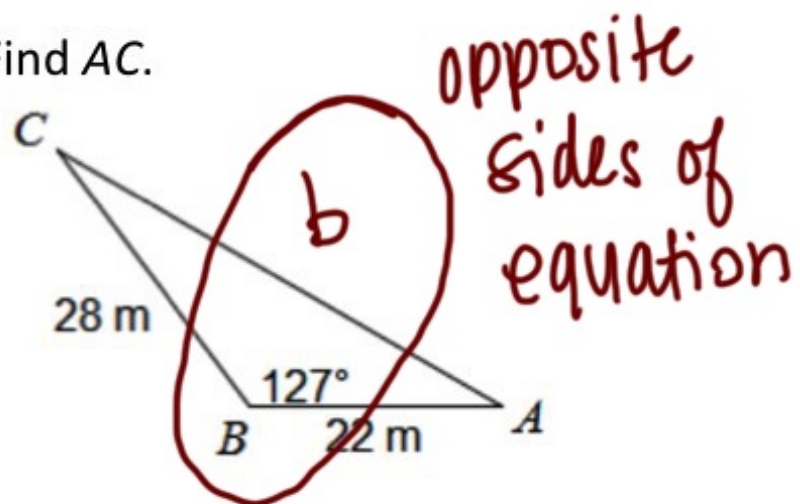
$$b^2 = 28^2 + 22^2 - 2(28)(22)\cos 127°$$

$$\sqrt{b^2} = \sqrt{2009.44}$$

$$\boxed{b = 44.83 \text{ m}}$$

P. Find *AB*.

B 10.4 cm

32.5°

A

29.3 cm

C

Q. Find *DF*.

D

8 yd

F   7 yd   4°   E

SKIP!!!

R. Find $m\angle C$.

B

a 6 km   7 km c

C   8 km   A

b
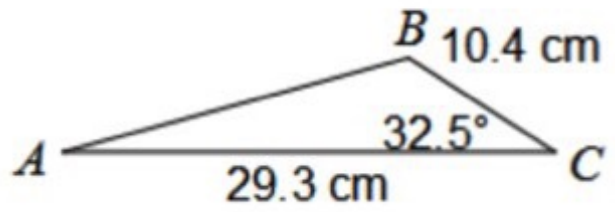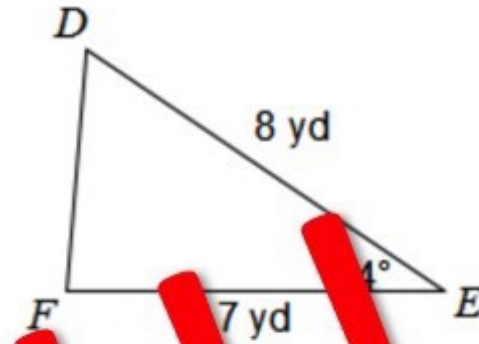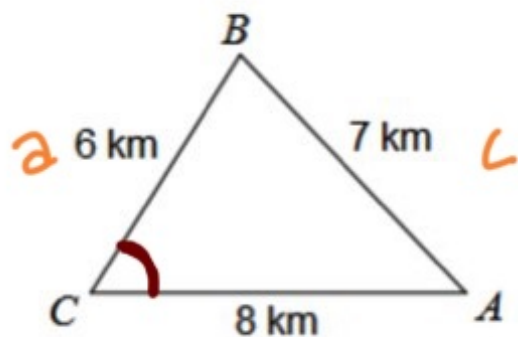
$7^2 = 6^2 + 8^2 - 2(6)(8) \cos C$

$49 = 100 - 96 \cos C$

$-100 \quad -100$

$\dfrac{-51}{-96} = \dfrac{-96 \cos C}{-96}$

$\cos^{-1}(\cos C) = (0.531)$ → $\boxed{C = 58°}$

S. Find $m\angle A$.

30 km
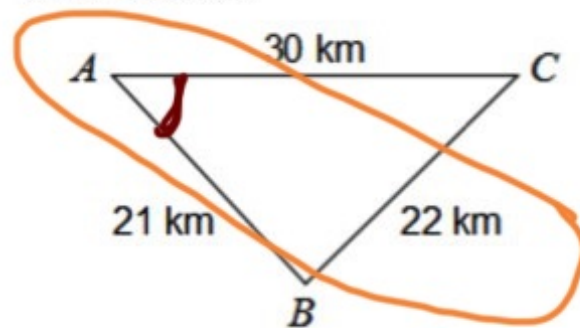
A   C   **opposite sides of the equation**

21 km   22 km

B

$22^2 = 21^2 + 30^2 - 2(21)(30) \cos A$

$984 = 1341 - 1260 \cos A$

$-1341 \quad -1341$
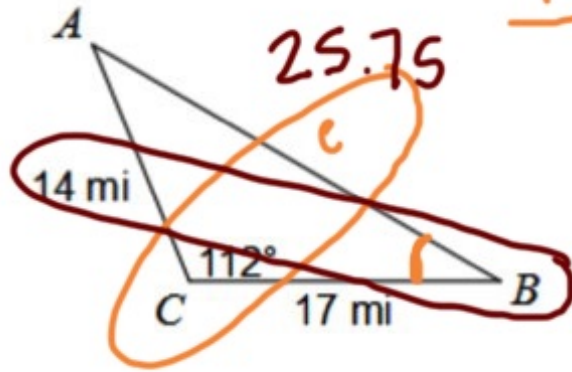
$\dfrac{-857}{-1260} = \dfrac{-1260 \cos A}{-1260}$

$\cos^{-1}(\cos A) = (0.680)$

$\boxed{A = 47°}$

Use the Law of Sines AND the Law of Cosines to find each measure indicated. If there is not enough information, write NEI.

T. Find $m\angle B$.

A

25.75

c

14 mi

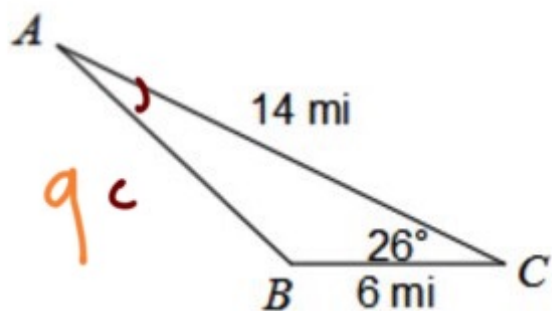112°

C   17 mi   B

Find c w/ law of cosines

$c^2 = 14^2 + 17^2 - 2(14)(17)\cos 112°$

$\sqrt{c^2} = \sqrt{663.31}$

$c = 25.75$ mi

Find $\angle B$ using Law of Sines

$14 \cdot \dfrac{\sin B}{14} = \dfrac{\sin 112°}{25.75} \cdot \dfrac{14}{1}$

$\sin^{-1}(\sin B) = \sin^{-1}(0.504) \rightarrow \boxed{B = 30°}$

U. Find $m\angle A$.

Use cosines to find side c

$$c^2 = 14^2 + 6^2 - 2(14)(6)\cos 26°$$

$$\sqrt{c^2} = \sqrt{81.00}$$

$$c = 9.00 \text{ mi}$$

Find $\angle A$ (sines)

$$6. \quad \frac{\sin A}{6} = \frac{\sin 26°}{9} \cdot \frac{6}{1}$$

$$\sin^{-1}(\sin A) \overset{\sin^{-1}}{=} \boxed{0.292}$$

$$\boxed{A = 16.98 \text{ mi}}$$

LAW OF SINES	LAW OF COSINES
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\bigg|$ $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$	$c^2 = a^2 + b^2 - 2ab\cos C$
Use when: ASA AAS SSA	Use when: SAS SSS