9.2 Angles and the Unit Circle

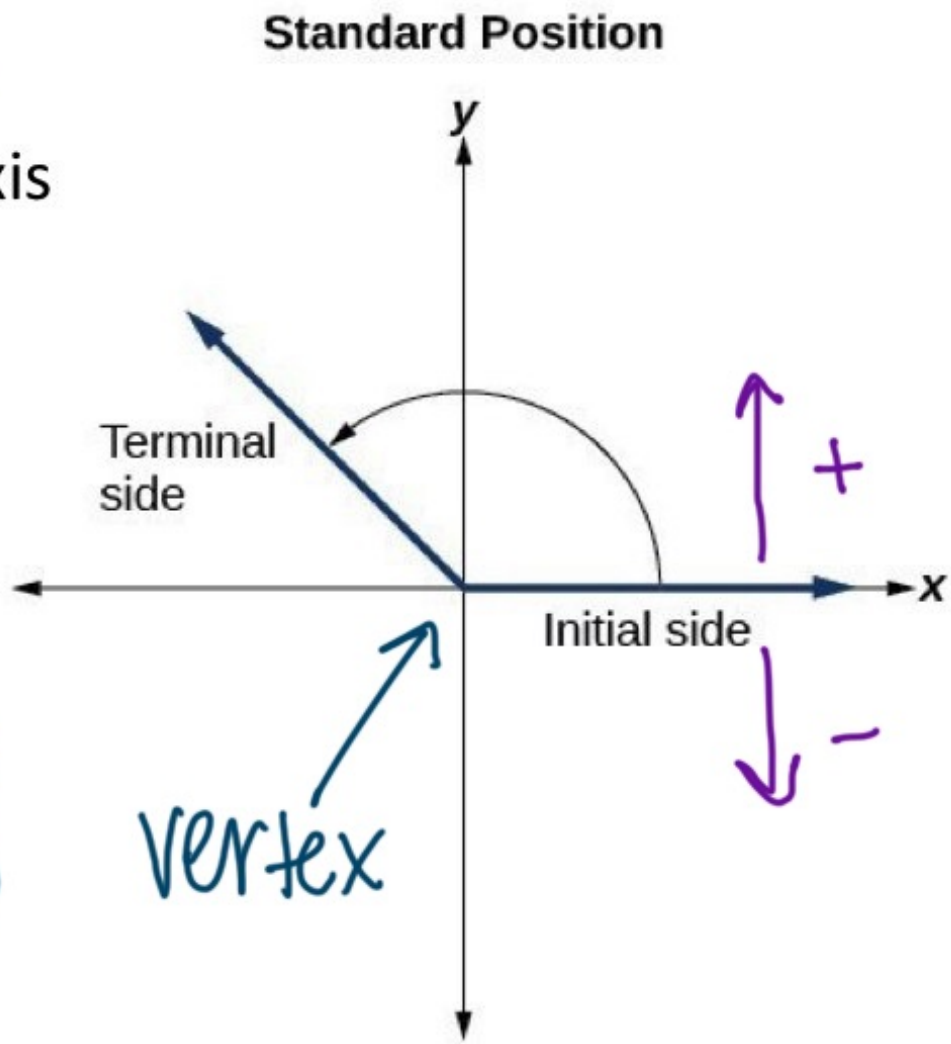
Standard Position: An angle is in standard position when the vertex is at the point (0,0)

Initial Side: The ray on the positive x-axis

Terminal Side: a ray that Rotate from the initial side

Positive Angles rotate counter clockwise (opens up)

Negative Angles rotate clockwise (opens down)

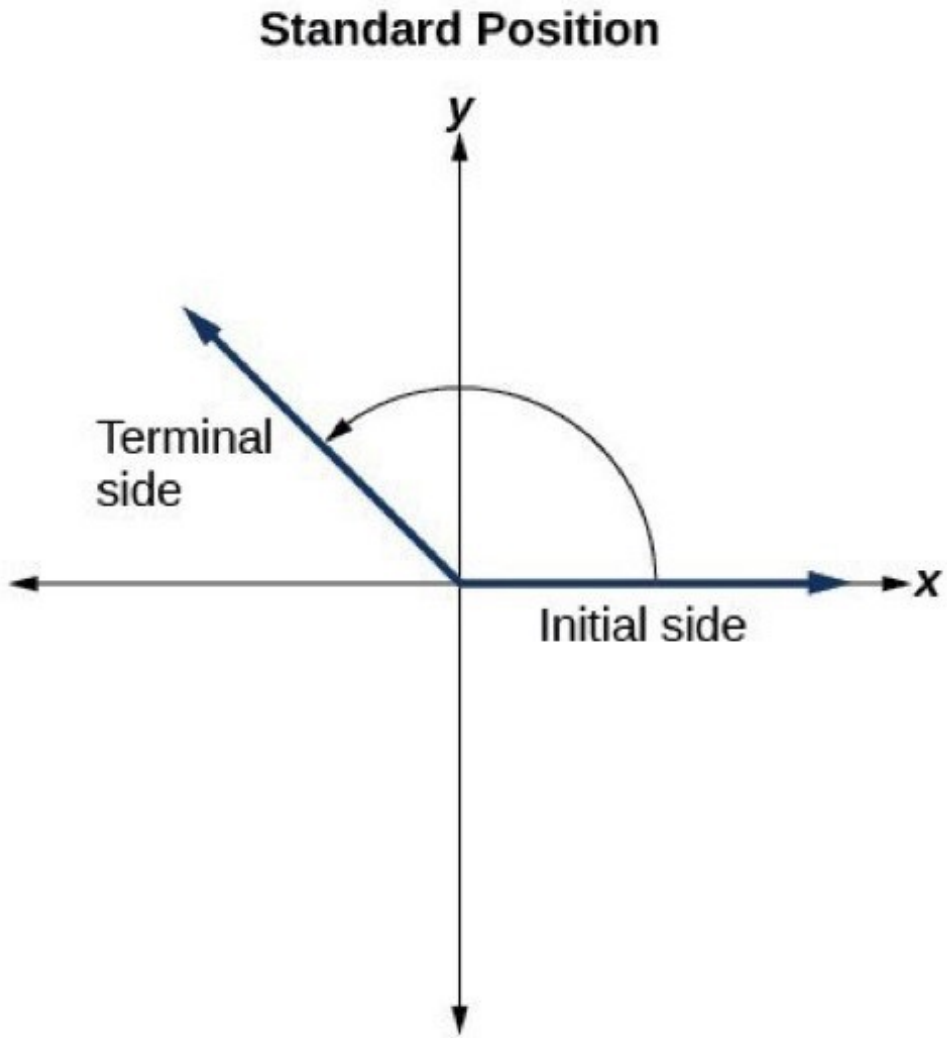


How many degrees make up a full rotation?

360°

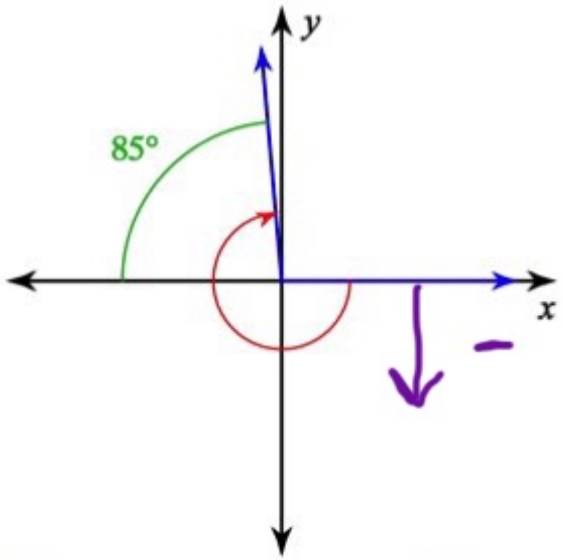
How many degrees make up a half rotation?

180°



EXAMPLE **Determine the measure of each angle.**

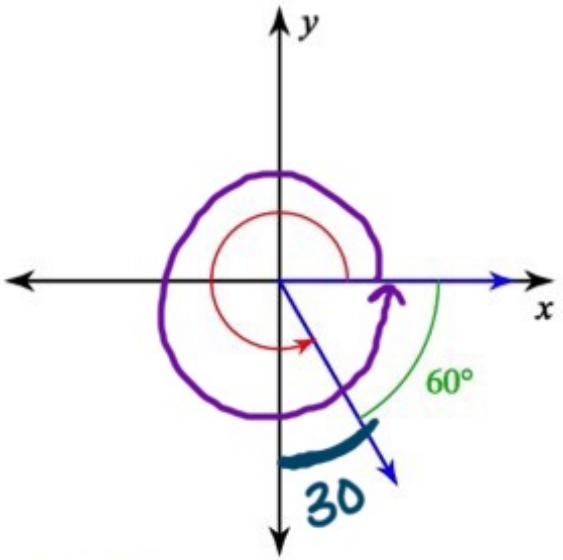
A.



$180 + 85 = 265$

$-265°$

B.

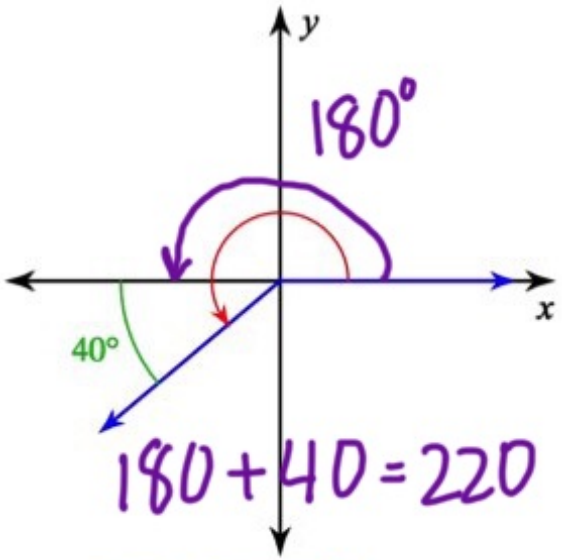


$360° - 60° = 300°$

OR $270° + 30° = 300°$

$300°$

C.



$180 + 40 = 220$

$220°$

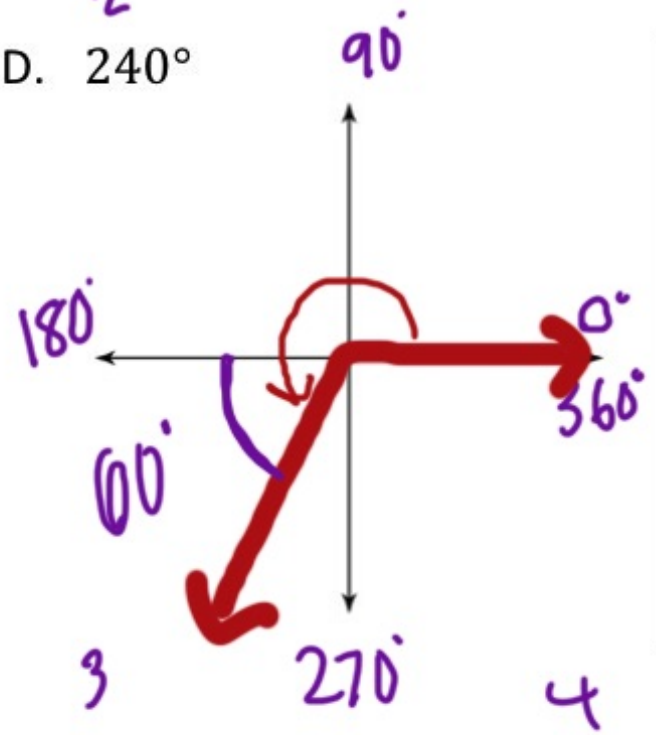
EXAMPLE

Draw each angle in standard position. What quadrant is the terminal side in?

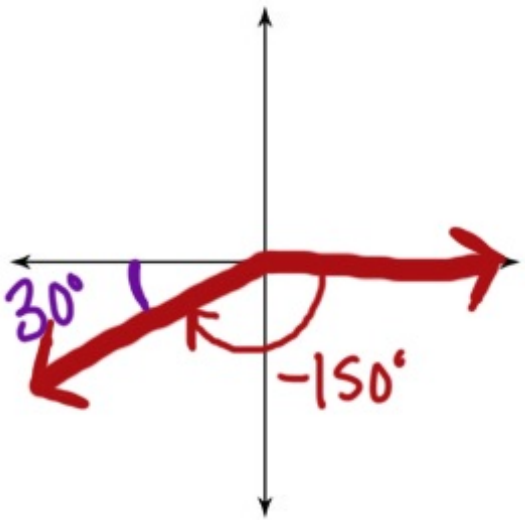
- show direction w/ arrow
- label closest angle to x-axis

2 1

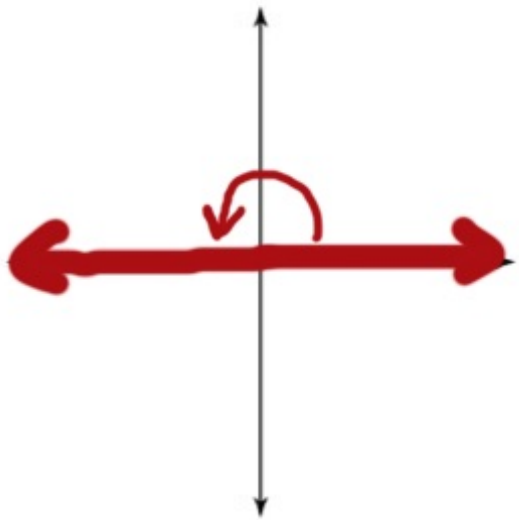
D. $240°$



E. $-150°$

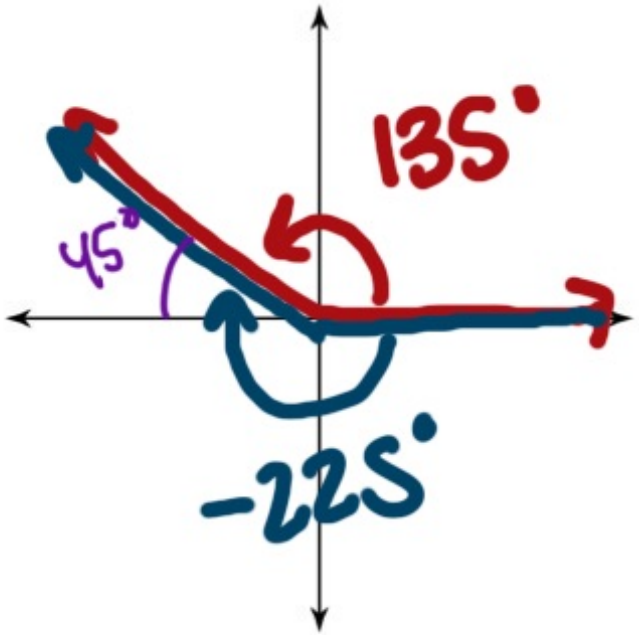


F. $180°$



EXAMPLE

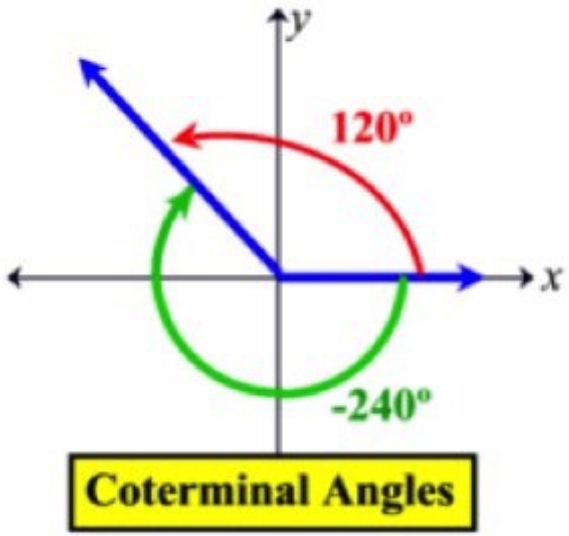
G. Draw 135° and –225° on the same coordinate plane. What do you notice?



Same!

Coterminal angle are angles in standard position with the same terminal side.

Strategy for finding coterminal angles	$\pm 360^\circ$



EXAMPLE

List three coterminal angles for each of the following. Make sure at least one of them is negative.

H. $60°$ $-360 = \boxed{-300°}$

$60+360 = \boxed{420°}$

$420+360 = \boxed{780°}$

OR $-300-360$

$\boxed{-660°}$

I. $500° \pm 360$

$140°$ $860°$

$-220°$ $1220°$

$-580°$ etc.

$-940°$

J. $-25° \pm 360$

$335°$ $-385°$

$695°$ $-1105°$

$1055°$ $-1465°$

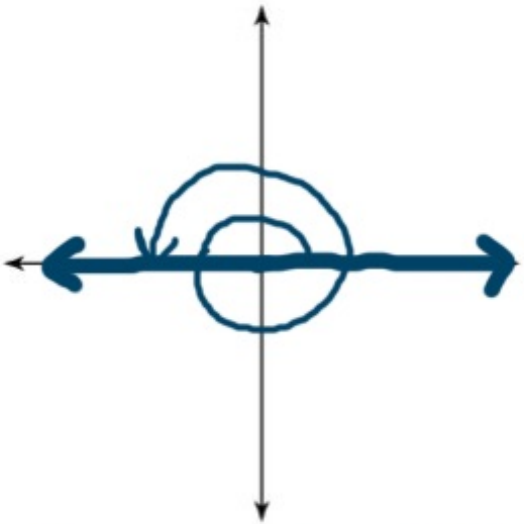
etc.

EXAMPLE

Draw each angle on the coordinate plane.

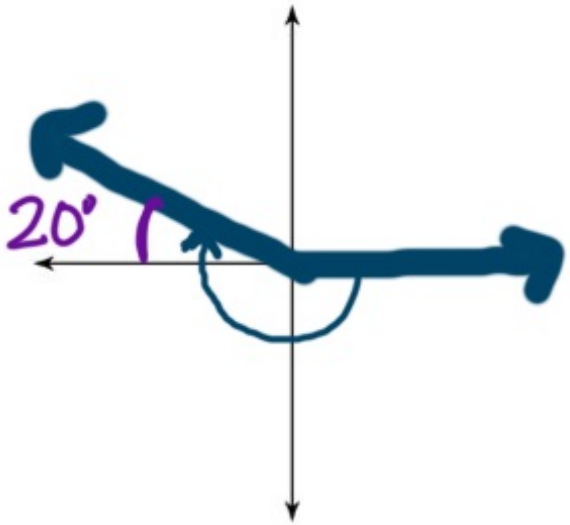
*find coterminal angle between 0 & 360°

K. 540°



540 − 360 = 180°

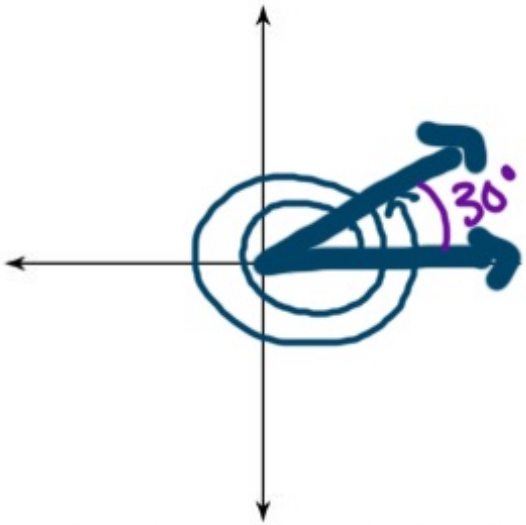
L. −200°



−200 + 360 =

160°

M. 750°



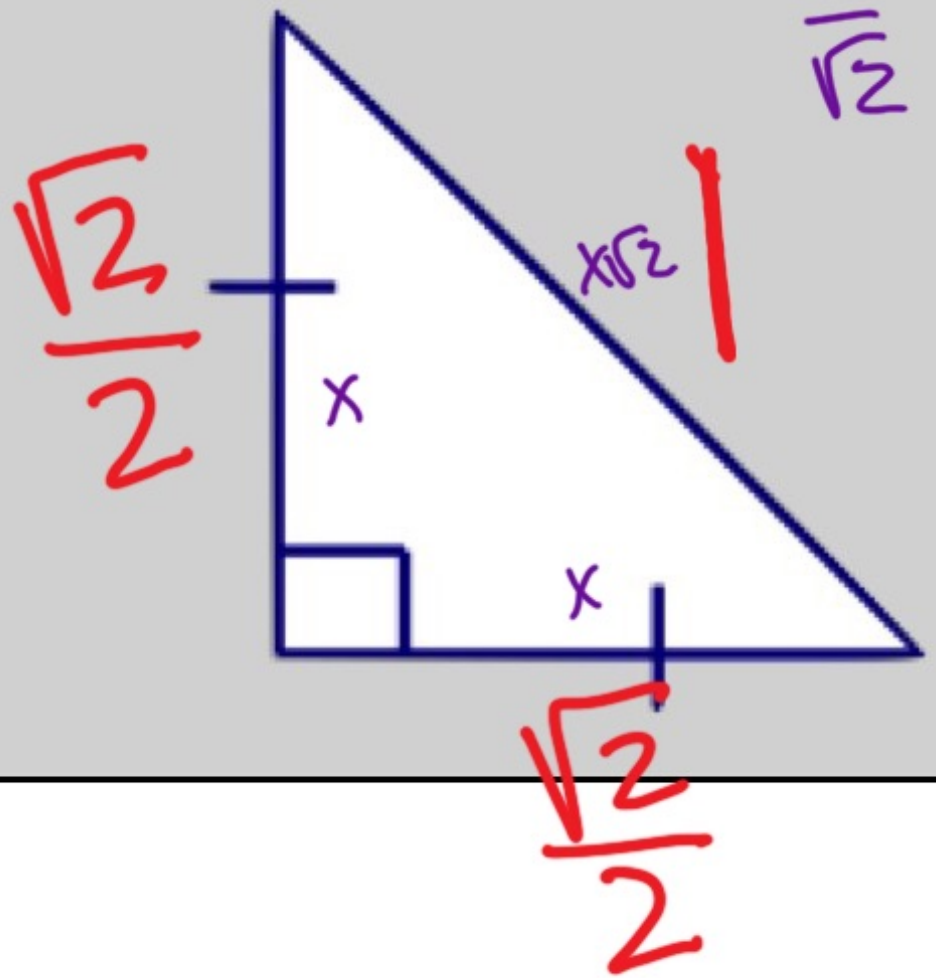
750 − 360 = 390°

390 − 360 = 30°

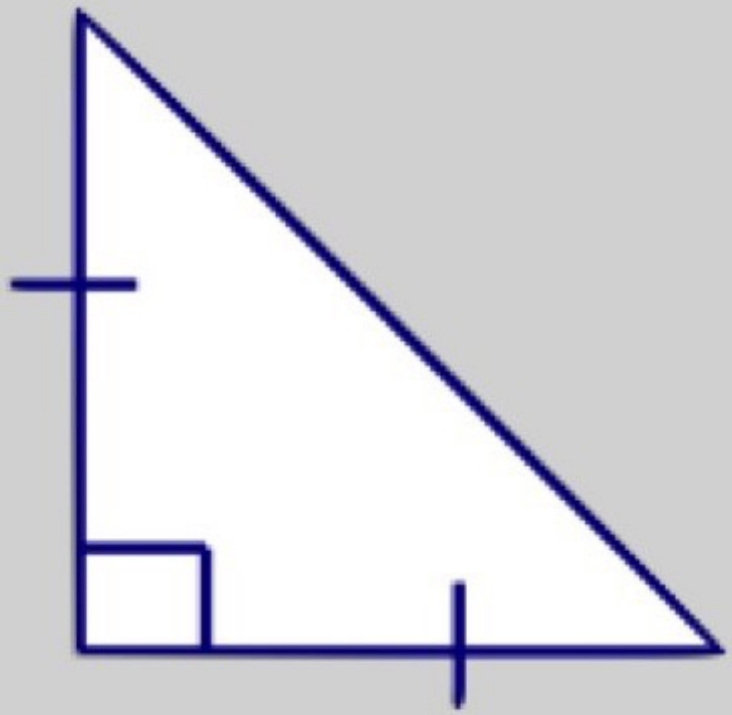
45°-45°-90° Triangle

$\frac{1}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \rightarrow x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

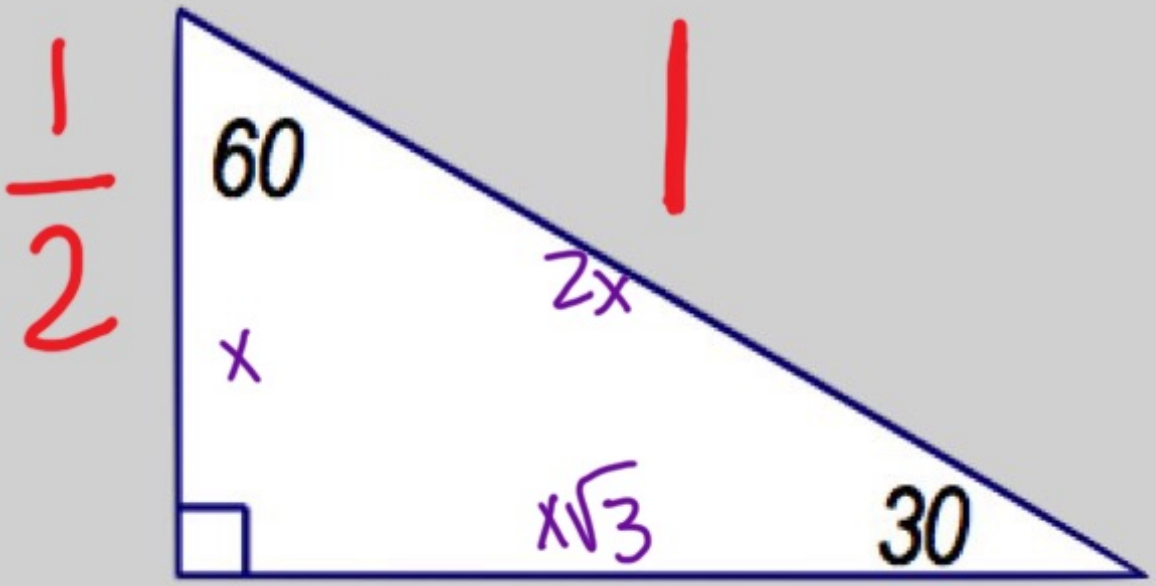
$x = \frac{\sqrt{2}}{2}$



45°-45°-90° Triangle



30°-60°-90° Triangle



$\frac{1}{2}$

1

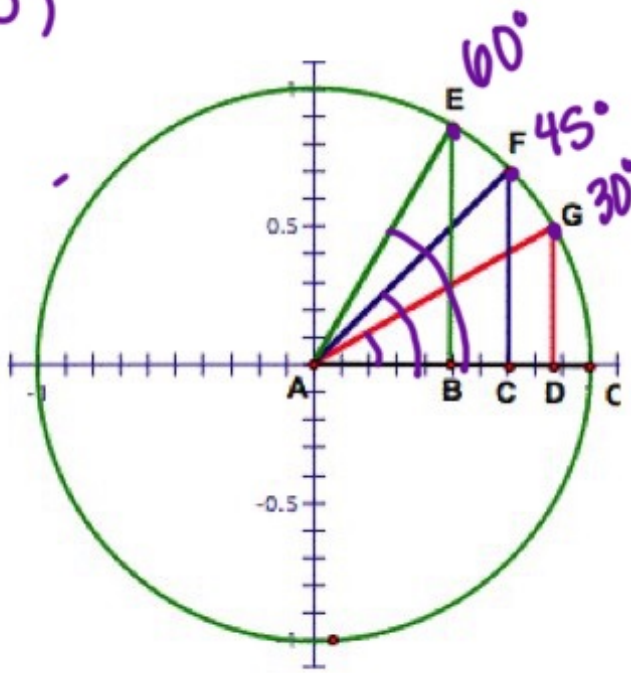
$\frac{1}{2} \cdot \frac{\sqrt{3}}{1}$

$\frac{\sqrt{3}}{2}$

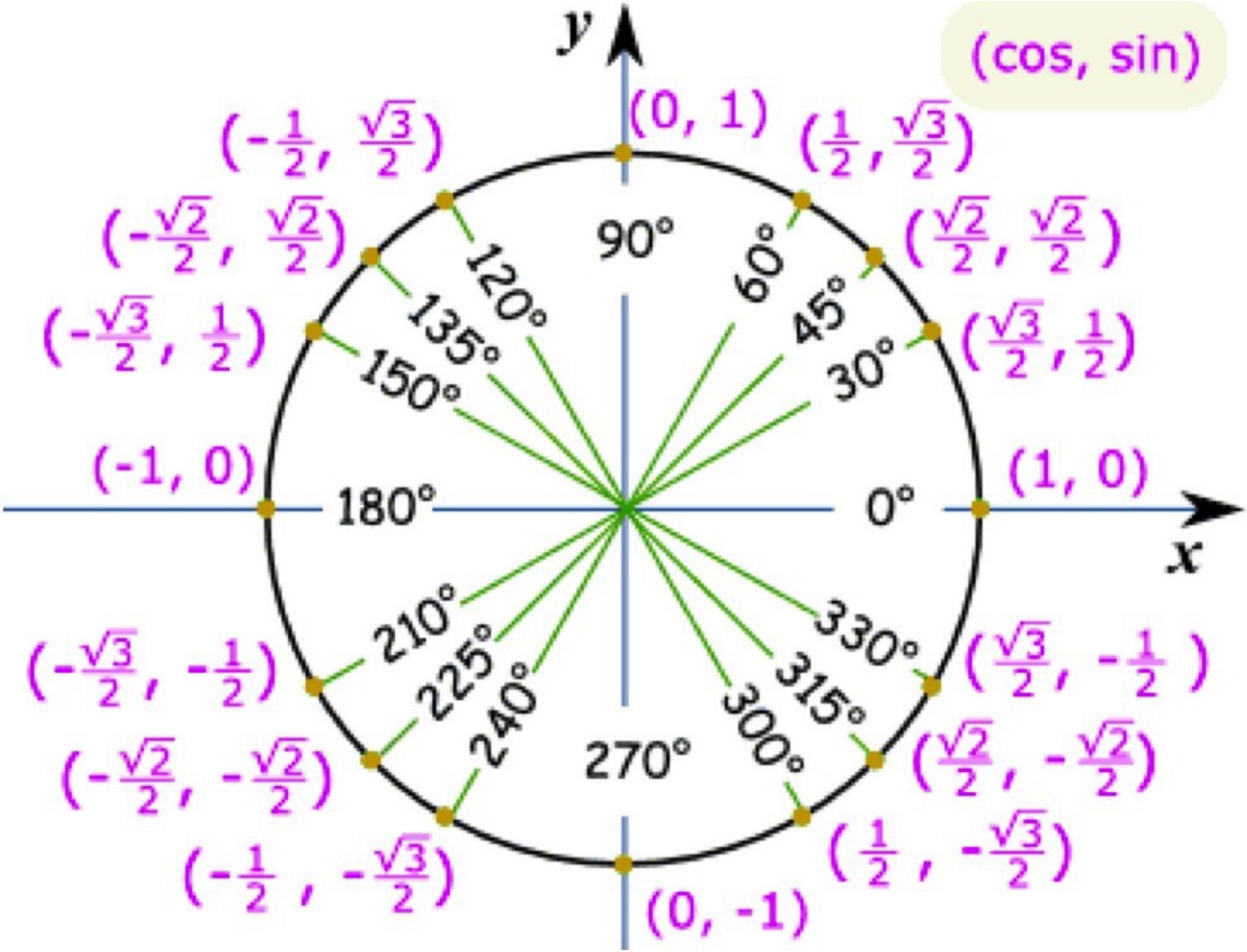
center: (0,0)

The Unit Circle

The Unit Circle is a circle with a radius of 1. It is segmented into 3 different triangles per quadrant, with interior angles of 30°, 45°, and 60°. The figure to the right illustrates this concept for the first quadrant.



We can use the relationships above to fill out the missing information for the unit circle below.


(cos, sin)
y
x
(0, 1)
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(1, 0)
(√3/2, -1/2)
(√2/2, -√2/2)
(1/2, -√3/2)
(0, -1)
(-1/2, √3/2)
(-√2/2, √2/2)
(-√3/2, 1/2)
(-1, 0)
(-√3/2, -1/2)
(-√2/2, -√2/2)
(-1/2, -√3/2)
90°
60°
45°
30°
0°
330°
315°
300°
270°
240°
225°
210°
180°
150°
135°
120°

$\sin\theta =$ y	$\cos\theta =$ x	$\tan\theta =$ y ÷ x

EXAMPLE

Find the EXACT values of the following trig functions:

N. $\sin 120°$

y

$\frac{\sqrt{3}}{2}$

O. $\cos 270°$

x

0

P. $\tan 45°$

y ÷ x

$\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = 1$

Q. $\tan 330°$

y ÷ x

$\frac{1}{2} \div \frac{\sqrt{3}}{2}$

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

R. $\sin 135°$

y

$\frac{\sqrt{2}}{2}$

S. $\cos 150°$

x

$-\frac{\sqrt{3}}{2}$