10.3 Period and Phase Shift

(OR COS)

$$y = a\sin(b(\theta - h)) + k$$

Amplitude

distance from midline to top or bottom

→ Vertical stretch

Frequency

$$\text{Period} = \frac{2\pi}{b}$$

# of times the graph repeats every 2π

Phase Shift

Horizontal shift

+ : left

− : Right

Midline

$y = k$

(Vertical shift)

+ : up

− : down

A. $y = 2\sin\left(3\left(\theta - \frac{\pi}{4}\right)\right) + 1$

Amplitude: 2

Frequency: 3

Period: $\frac{2\pi}{3}$

Phase shift: Right $\frac{\pi}{4}$

Midline: $y = 1$

B. $y = -\cos\left(\theta + \frac{\pi}{6}\right) - 4$

Amplitude: 1

Frequency: 1

Period: 2π

Phase shift: left $\frac{\pi}{6}$

Midline: $y = -4$

C. $y = -3\cos\left(\frac{1}{4}(\theta + \pi)\right) + 0$

Amplitude: 3

Frequency: $\frac{1}{4}$

Period: $2\pi / \frac{1}{4} = \frac{2\pi}{1} \cdot \frac{4}{1} = 8\pi$

Phase shift: left π

Midline: $y = 0$

D. $y = 2\cos\left(4\left(\theta - \frac{\pi}{2}\right)\right) - 2$

Amplitude: 2

Frequency: 4

Period: $2\pi/4 = \pi/2$

Phase shift: Right $\pi/2$

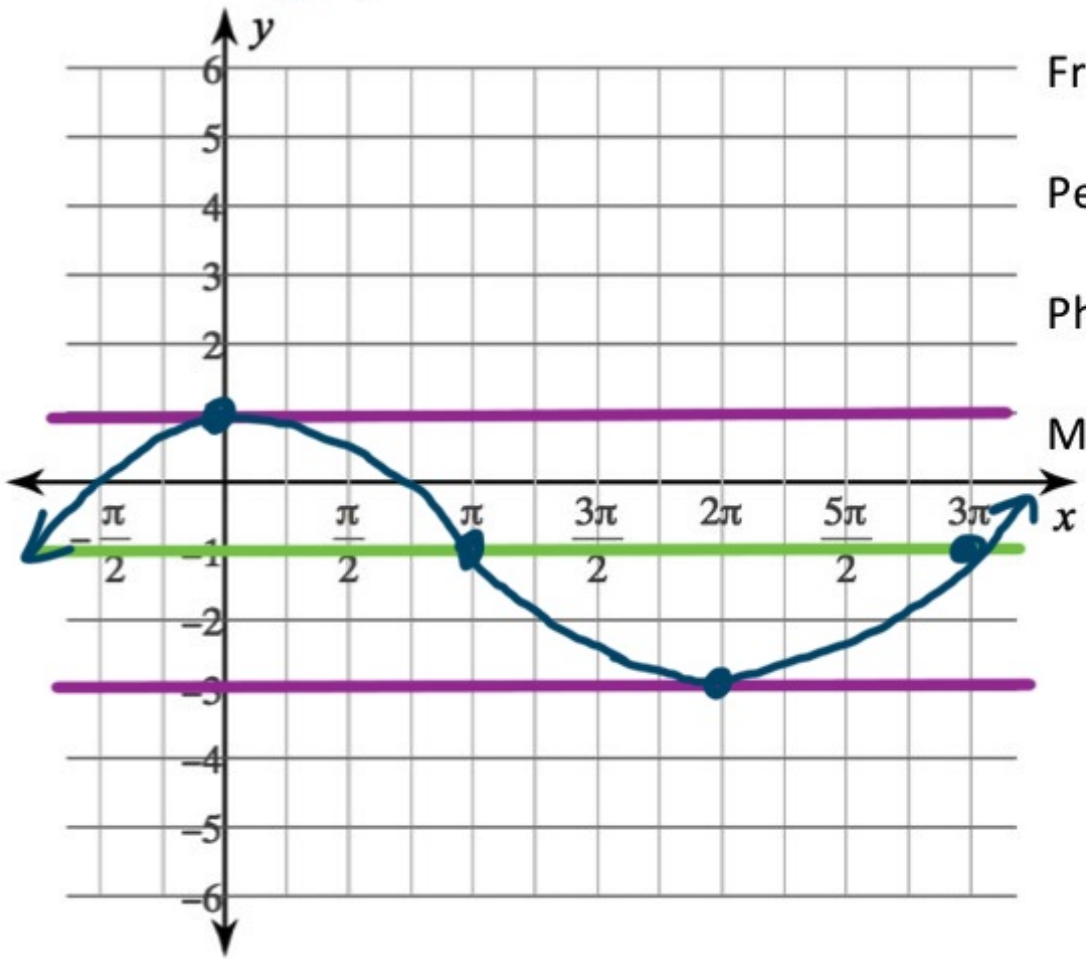
Midline: $y = -2$

Steps to graphing:

1. Identify the parent function. ($\sin\theta$ or $\cos\theta$)
2. Determine the amplitude, midline, period, and phase shift.
3. Mark the midline and phase shift on the graph (this is your new x- and y-axis).
4. Mark the 5-point pattern on the graph $\left(\frac{period}{4}\right)$. mark the amplitude
5. Sketch pretty WAVE.

(on pg. 8 of the notes)

E. $y = 2\cos\left(\frac{1}{2}\theta\right) - 1$



Amplitude: 2

Frequency: $\frac{1}{2}$

Period: $2\pi / \frac{1}{2} = 2\pi \cdot \frac{2}{1} = 4\pi$

Phase shift: None

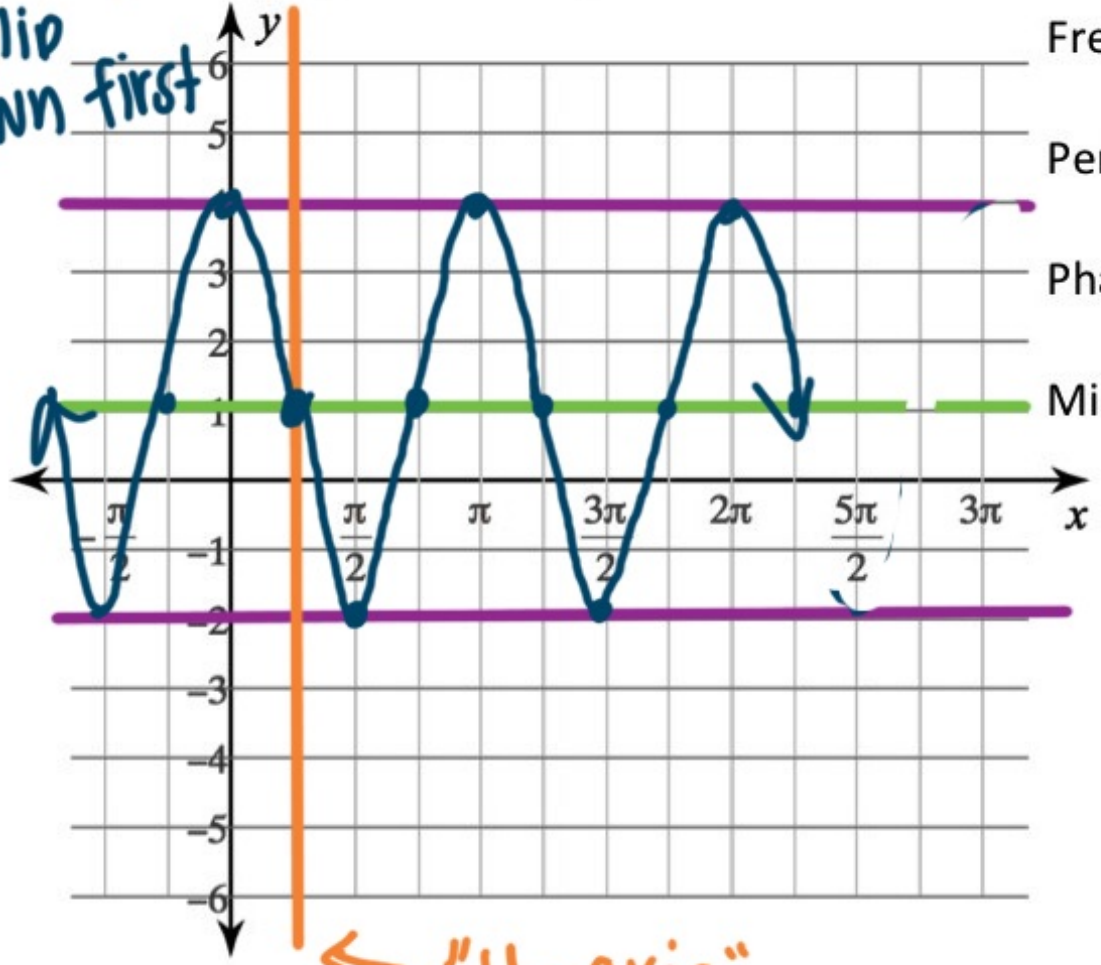
Midline: $y = -1$

Pattern: $\frac{4\pi}{4}$

Period

F. $y = -3\sin\left(2\left(\theta - \frac{\pi}{4}\right)\right) + 1$

flip - down first



← "y-axis"

starting point

Amplitude: 3

Frequency: 2

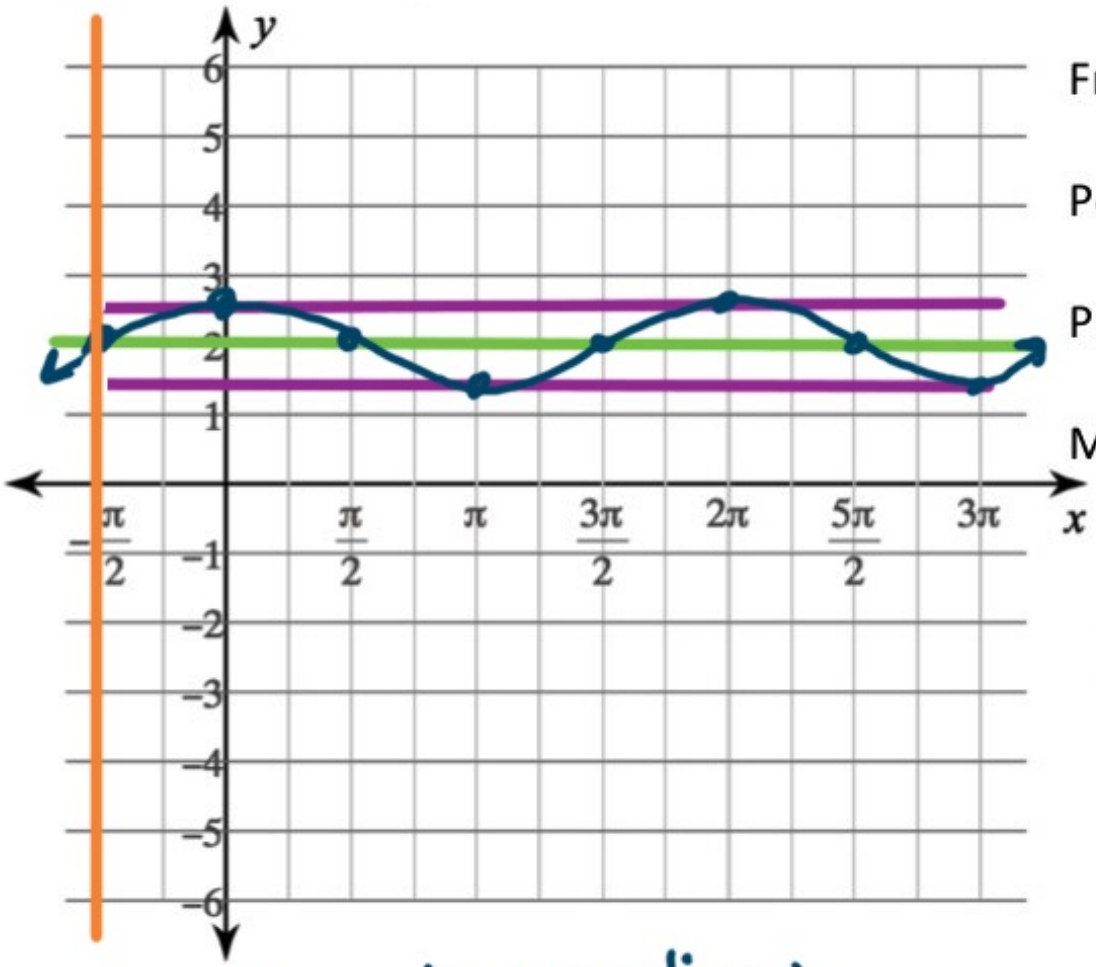
Period: $2\pi/2 = \pi$

Phase shift: Right $\pi/4$

Midline: $y = 1$

pattern: $\frac{\pi}{4}$ ← period

G. $y = \frac{1}{2}\sin\left(\theta + \frac{\pi}{2}\right) + 2$



Amplitude: 1/2

Frequency: 1

Period: $2\pi/1 = 2\pi$

Phase shift: left $\pi/2$

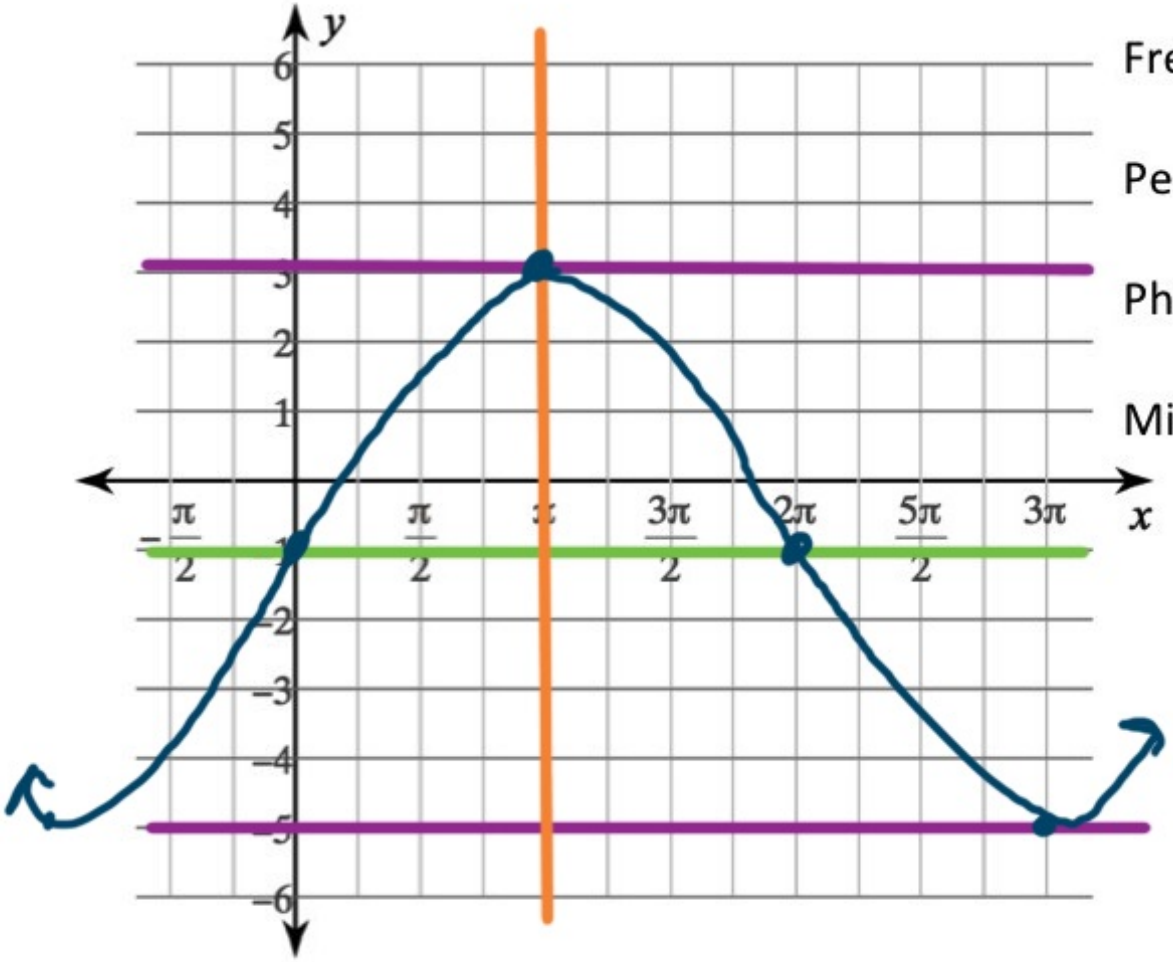
Midline: $y = 2$

Pattern: $\frac{2\pi}{4} = \frac{\pi}{2}$

Alternate equation:

$y = \frac{1}{2}\cos\theta + 2$

H. $y = 4\cos\left(\frac{1}{2}(\theta - \pi)\right) - 1$



Amplitude: 4

Frequency: $\frac{1}{2}$

Period: $2\pi / \frac{1}{2} = 4\pi$

Phase shift: Right π

Midline: $y = -1$

Pattern: $\frac{4\pi}{4} = \pi$

I. The cosine function that has been translated 2 units down and 5 units to the left

midline: $y = -2$

phase shift: left 5

$$y = \cos(\theta + 5) - 2$$

J. A sine function with an amplitude of 3 and a period of $\frac{\pi}{2}$ that has been shifted down 2 units and right 3 units

amp: 3

per: $\frac{\pi}{2}$ → frequency: 4

midline: $y = -2$

phase shift: right 3

$$y = 3\sin(4(\theta - 3)) - 2$$