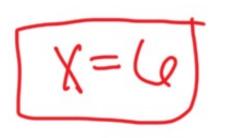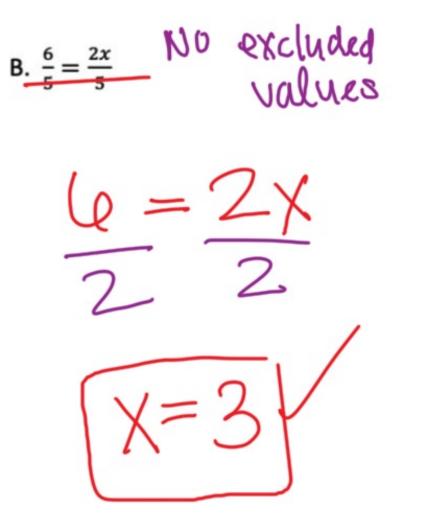# 6.4 Solving Rational Equations

A. What are the solutions of the rational equation?

$$\frac{2}{2}\frac{3}{4} = \frac{x}{8}$$ No excluded values

$$\frac{6}{8} = \frac{x}{8}$$

$$\boxed{x = 6}$$

excluded values

STEP 1: Find domain of all fractions (need to factor denominators)

STEP 2: Get LCD (least common denominator) by multiplying each fraction by the factor that it's missing.

STEP 3: Cancel denominators and solve.

STEP 4: Check solutions for domain.

B. $\dfrac{6}{5} = \dfrac{2x}{5}$ NO excluded values

$$\dfrac{6}{2} = \dfrac{2x}{2}$$

$$\boxed{x = 3}$$

C. $\dfrac{2}{2}\dfrac{3}{4} = \dfrac{x+1}{8}$ NO excluded values

$$6 = x + 1$$
$$-1 \qquad -1$$

$$\boxed{x = 5}$$

D. $\frac{6}{5} = \frac{x-5}{5}$ No excluded values

$6 = x - 5$
$+5 \qquad +5$

$\boxed{x = 11}$

E. $\frac{1}{x-2} = \frac{x+2}{5x-10}$

$\frac{5 \cdot 1}{5(x-2)} = \frac{x+2}{5(x-2)}$

$5 = x + 2$
$-2 \qquad -2$

$\boxed{x = 3}$ excluded values: $x \neq 2$

F. $\dfrac{8}{x+3} = \dfrac{x+1}{x+6}$

excluded values:
$x \neq -3, -6$

$\dfrac{(x+6)\,8}{(x+6)(x+3)} = \dfrac{(x+1)(x+3)}{(x+6)(x+3)}$

$8x + 48 = x^2 + 3x + x + 3$

$8x + 48 = x^2 + 4x + 3$
$-8x - 48 \qquad\qquad -8x - 48$

$0 = x^2 - 4x - 45$

$0 = (x-9)(x+5)$

$\boxed{x = 9, \; x = -5} \;\checkmark$

G. $\dfrac{x-1}{x+2} = \dfrac{x^2+2x-3}{x+2}$

excluded values:
$x \neq -2$

$x - 1 = x^2 + 2x - 3$
$-x + 1 \qquad\qquad -x + 1$

$0 = x^2 + x - 2 \qquad \dfrac{2}{2} \cdot \dfrac{-1}{-1} \begin{array}{c} -2 \\ 1 \end{array}$

$0 = (x+2)(x-1)$

$\cancel{x=-2} \quad \boxed{x=1}\;\checkmark$

$\dfrac{-9}{-9} \cdot \dfrac{5}{5} \begin{array}{c} -45 \\ -4 \end{array}$

extraneous solution:
a solution that is also an excluded value
$\rightarrow$ Throw it out!

4. $\dfrac{1}{x+4} - \dfrac{3}{x} = \dfrac{2}{x+4}$

1. $\dfrac{x^2-29}{x^2-10x+21} = \dfrac{6}{x-7} + \dfrac{5}{x-3}$

excluded values:
$x \neq 7, 3$

$\dfrac{-3}{-3} \cdot \dfrac{-7}{-7} \quad 21$
$\underline{-3 + -7 \, -10}$

$\dfrac{x^2-29}{(x-3)(x-7)} = \dfrac{6}{(x-7)}\dfrac{(x-3)}{(x-3)} + \dfrac{5}{(x-3)}\dfrac{(x-7)}{(x-7)}$

$x^2 - 29 = 6x - 18 + 5x - 35$

$x^2 - 29 = 11x - 53$

$-11x + 53 \quad -11x + 53$

$x^2 - 11x + 24 = 0$

$\dfrac{-8}{-8} \cdot \dfrac{-3}{-3} \quad 24$
$\underline{-8 + -3 \, -11}$

$(x-8)(x-3) = 0$

$\boxed{x = 8}, \quad x = \cancel{3} \leftarrow \text{extraneous}$

Skip J and K

Setup for Work Problems:

$$\frac{1}{time 1} + \frac{1}{time 2} = \frac{1}{time\ together}$$

L. You can wash one window in 15 minutes and your sister can wash one window in 20 minutes. How many minutes will it take to wash 12 windows if you work together?

$$\frac{4x}{4x} \cdot \frac{1}{15} + \frac{3x}{3x} \cdot \frac{1}{20} = \frac{1}{x} \cdot \frac{60}{60}$$

$$4x + 3x = 60$$

$$\frac{7x}{7} = \frac{60}{7}$$

$$x = \frac{60}{7} \approx 8.57 \text{ min} \leftarrow 1 \text{ window}$$

$$\frac{12 \text{ windows}}{(8.57)12 = \boxed{102.86 \text{ min}}}$$

6.4

HW: 2-12 even, 13, 16

M. One pump can fill a water tank twice as fast as a second pump. Working together, the two pumps can fill the cistern in 5 hours. Find how long it takes each pump to fill the cistern when working alone.

Pump 1   Pump 2   together

$$\frac{10}{10} \cdot \frac{1}{x} + \frac{5}{5} \cdot \frac{1}{2x} = \frac{1}{5} \cdot \frac{2x}{2x}$$

Pump 1: 7.5 hr.

Pump 2: 15 hr.

$$10 + 5 = 2x$$

$$\frac{15}{2} = \frac{2x}{2} \rightarrow x = 7.5 \text{ hr.}$$