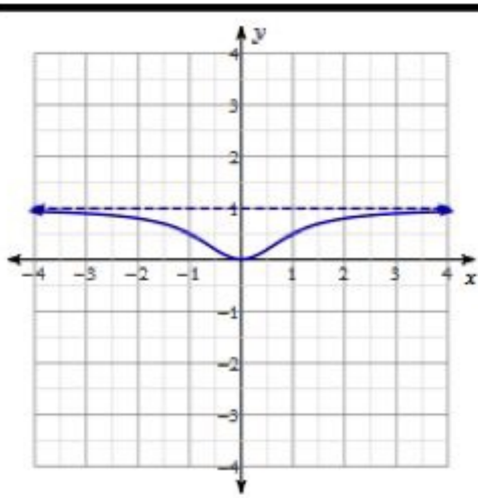
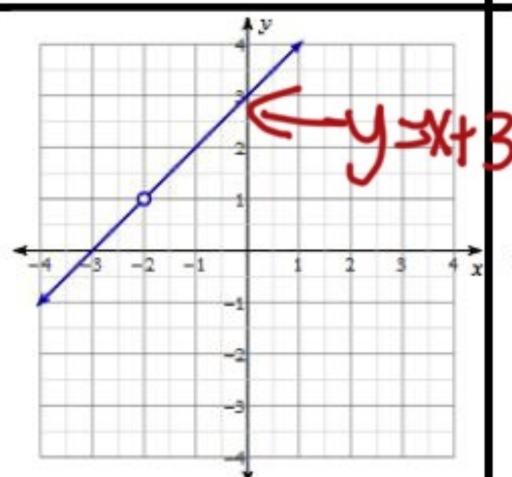
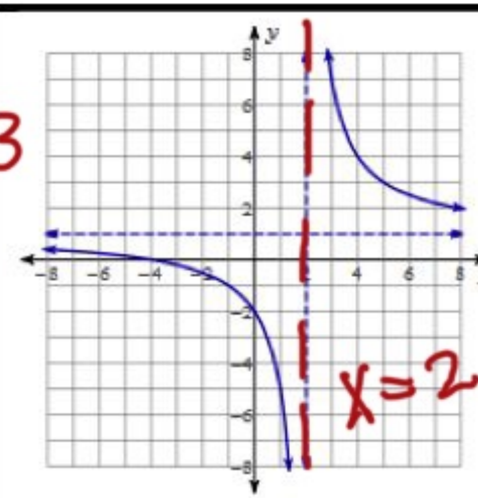
5.1 Rational Graph Properties

Find the domain for each of the rational functions below.

X

$x^2+1=0$
$-1 \quad -1$
$x^2=-1$

$y=\dfrac{x^2}{x^2+1}$	$y=\dfrac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}}$ $=x+3$	$y=\dfrac{x+4}{x-2}$
Excluded Values: No excluded values	Excluded Values: $x \neq -2$	Excluded Values: $x \neq 2$
Domain: $(-\infty,\infty)$	Domain: $(-\infty,-2)\cup(-2,\infty)$	Domain: $(-\infty,2)\cup(2,\infty)$
Continuous Graph No excluded values → No breaks	Removable Discontinuity one or more factors cancel → hole	Non-Removable Discontinuity Factor stays in the denominator → Vertical Asymptote
		

A discontinuity is basically a break in the graph/domain

How do you find discontinuities?

excluded values (set denominator = 0)

Holes (point)

To find a hole (Removeable) we need to make sure that the function is simplified, meaning we need to factor & cancel.

- The *x*-coordinate is the zero from any factor that cancelled out. → excluded value
- To find the *y*-coordinate, we plug the *x*-coordinate into the simplified function.

Ex: $f(x) = \frac{(x+5)\cancel{(x+2)}}{\cancel{x+2}}$

$f(x) = x+5,\ x \neq -2$

$f(-2) = -2+5 = 3$

$\boxed{\text{hole: } (-2,3)}$

EXAMPLE

State the domain for each function. State any discontinuities and identify any asymptotes.

A. $y = \frac{x+3}{x^2-4x+3}$

Domain:

Holes:

VA:

HA:

Vertical Asymptotes

Vertical asymptotes (non removable) appear in the denominator of a **simplified** rational function. These vertical lines are written in the form $x=k$, where k is a constant.

- To find the location of the asymptote, find the x-values that make the denominator equal to 0. * excluded value that WAS NOT cancelled out

Ex: $f(x) = \frac{1}{x-2}$, $x \neq 2$

$\boxed{\text{VA: } x=2}$

End Behavior

Horizontal (Behavioral) Asymptotes

To find a horizontal asymptote of a graph of a rational function, compare the degree of the numerator (m) with the degree of the denominator (n).

- If $m < n$ (bottom heavy), the graph has horizontal asymptote at $y=0$.
- If $m > n$ (top heavy), the graph has no horizontal asymptote.
- If $m = n$ (equally weighted), you get the horizontal asymptote by dividing the leading coefficient of the numerator (a) by the leading coefficient of the denominator (b).

$y=\frac{a}{b}$

Ex: $f(x) = \frac{1x^2}{3x^2-12}$

equally weighted

$\boxed{\text{HA: } y=\frac{1}{3}}$

Ex: $f(x) = \frac{x+1}{2x^2+2}$

bottom heavy

$\boxed{\text{HA: } y=0}$

EXAMPLE

State the domain for each function. State any discontinuities and identify any asymptotes.

$-3 \times -1 \quad 3$
$-3 + -1 \quad -4$

A. $y = \frac{x+3}{x^2-4x+3} = \frac{x+3}{(x-3)(x-1)}$; $x \neq 1, 3$

Domain:

$(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

Holes:

None

VA:

$x = 1 \quad x = 3$

HA:

$y = 0$

B. $y = \frac{x-5}{x^2+1}$ (No excluded values)

Domain:

$(-\infty, \infty)$

Holes:

None

VA:

None

HA:

$y = 0$

C. $y = \frac{x^2-3x-4}{x-4}$

$-4 \times 1 = -4$
$-4 + 1 = -3$

$\frac{\cancel{(x-4)}(x+1)}{\cancel{x-4}} = x+1, \ x \neq 4$

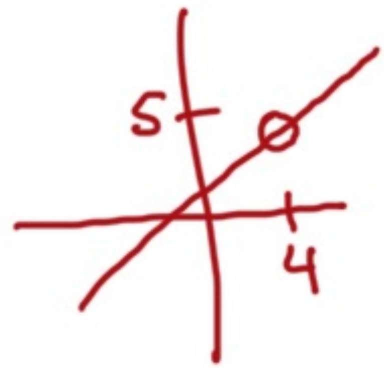
Domain:

$(-\infty, 4) \cup (4, \infty)$

Holes:

$x=4$

$f(4) = 4+1 = 5 \quad (4, 5)$



VA:

None

HA:

None

D. $y = \frac{1}{x^2-16}$ $= \frac{1}{(x-4)(x+4)}$, $x \neq 4, -4$

Domain:

$(-\infty,-4) \cup (-4,4) \cup (4,\infty)$

Holes:

None

VA:

$x=4$, $x=-4$

HA:

$y=0$

E. $y = \frac{x+1}{x^2-5x-6} = \frac{\cancel{x+1}}{\cancel{(x+1)}(x-6)} = \frac{1}{x-6}$, $x = -1, 6$

Domain:

$(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$

Holes:

$x = -1$

$f(-1) = \frac{1}{-1-6} = -\frac{1}{7}$ $\left(-1, -\frac{1}{7}\right)$ (excluded value that cancelled out)

VA:

$x = 6$ (excluded value from simplified version)

HA:

$y = 0$

F. $y = \frac{x^2+2x}{x+2}$ $\frac{x\cancel{(x+2)}}{\cancel{x+2}} = x,\ x \neq -2$

Domain:

$(-\infty, -2) \cup (-2, \infty)$

Holes:

$x = -2$

$f(-2) = -2$ $(-2, -2)$

VA:

None

HA:

None

To find *x*-intercepts:

Set the top of the SIMPLIFIED function equal to 0

To find *y*-intercepts:

plug 0 in for x $\left(\dfrac{\text{constant term of top}}{\text{constant term of bottom}}\right)$

EXAMPLE

Find all *x* and *y*-intercepts for each function.

G. $y = \frac{x-3}{x+5}$

$$(x+5)\,0 = \frac{x-3}{\cancel{x+5}} \cdot \cancel{x+5}$$

$$0 = x-3$$

x-intercept(s):

$x-3=0$ $(3,0)$

y-intercept:

$(0, -3/5)$ $f(0) = \frac{0-3}{0+5} = -\frac{3}{5}$

H. $y = \frac{x-3}{x^2+5x+6}$

$2 \cdot 3 = 6$
$2+3=5$

$\frac{x-3}{(x+2)(x+3)}$

x-intercept(s):

$x-3=0$ $\boxed{(3,0)}$

y-intercept:

$f(0) = -3/6 = -1/2$ $(0,-1/2)$

Find…	**By…**	
Discontinuities	setting the denominator equal to zero	
	Removable	Non-Removable
	Factors cancel $\rightarrow$ hole	Factors don't cancel $\rightarrow$ Vertical Asymptote
Holes	Plug the excluded value that cancelled out into the simplified function to find y	
Vertical Asymptotes	excluded values that didn't cancel out.	

<table>
<tr><td rowspan="3">Horizontal Asymptote</td><td colspan="2">look @ highest degree on top & bottom</td></tr>
<tr><td>Bottom Heavy</td><td>Equally Weighted</td></tr>
<tr><td>$y = 0$</td><td>Divide leading coefficients on top and bottom</td></tr>
<tr><td>x-intercepts</td><td colspan="2">Set numerator $= 0$ (simplified)</td></tr>
<tr><td>y-intercept</td><td colspan="2">Plug 0 in for x.</td></tr>
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