5.2 Graphing Rational Functions

You can get a reasonable graph for a rational function by finding all intercepts and asymptotes. Sometimes you will also have to plot a few extra points to get a good sense of the shape of the graph.

A. $y = \frac{1x^2+x-12}{1x^2-4}$

$-3 \times 4 = -12$, $-3 + 4 = 1$ $-2 \times 2 = -4$, $-2 + 2 = 0$

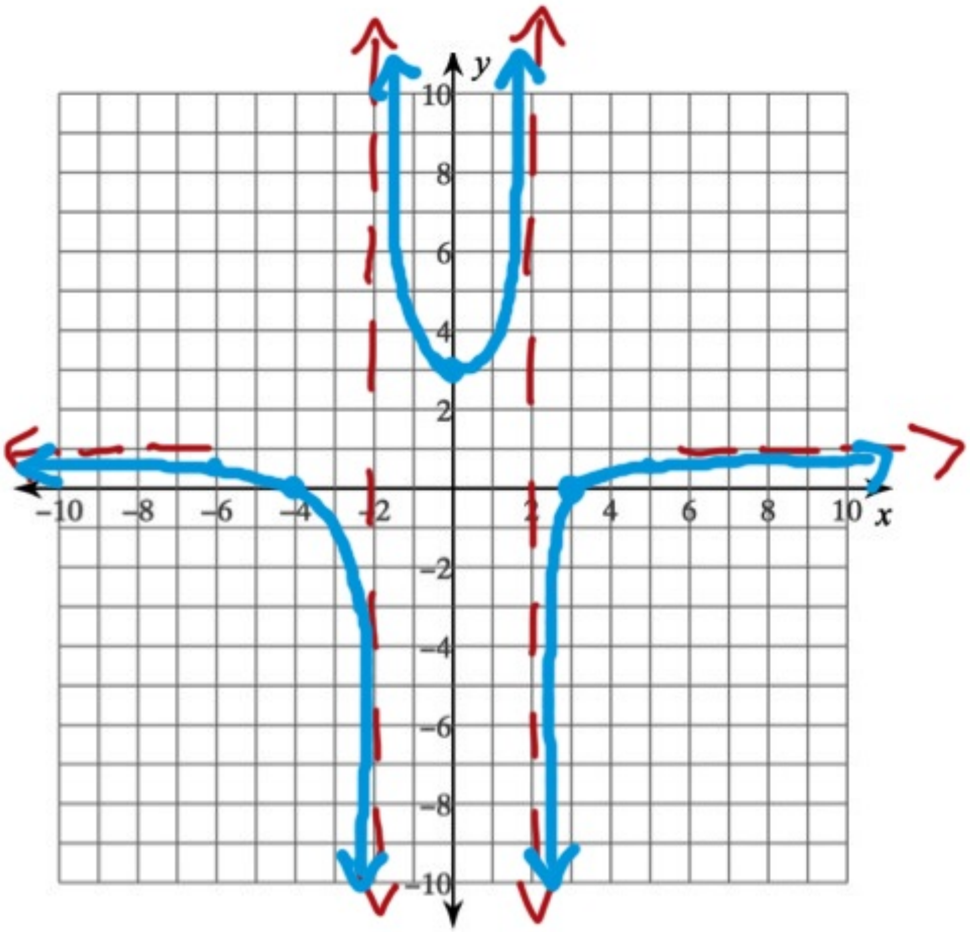
i) $y = 1$ (equally weighted)

ii) $\frac{(x-3)(x+4)}{(x-2)(x+2)}$, $x \neq 2, -2$ VA: $x = 2$
$x = -2$
no holes

iii) $x - 3 = 0$
$x + 4 = 0$
$\left.\begin{array}{l}(3,0) \\ (-4,0)\end{array}\right\}$ x-int.

y-int: $f(0) = \frac{-12}{-4} = 1$ $\boxed{(0,3)}$

A. Continued



ii) $f(-6) = \dfrac{(-6+4)(-6-3)}{(-6-2)(-6+2)}$

$= \dfrac{+18}{32} = 0.56$

$f(5) = \dfrac{(5-3)(5+4)}{(5-2)(5+2)}$

$= \dfrac{18}{21} = 0.86$

B. $y = \frac{4x}{x^3} = \frac{4}{x^2}, x \neq 0$

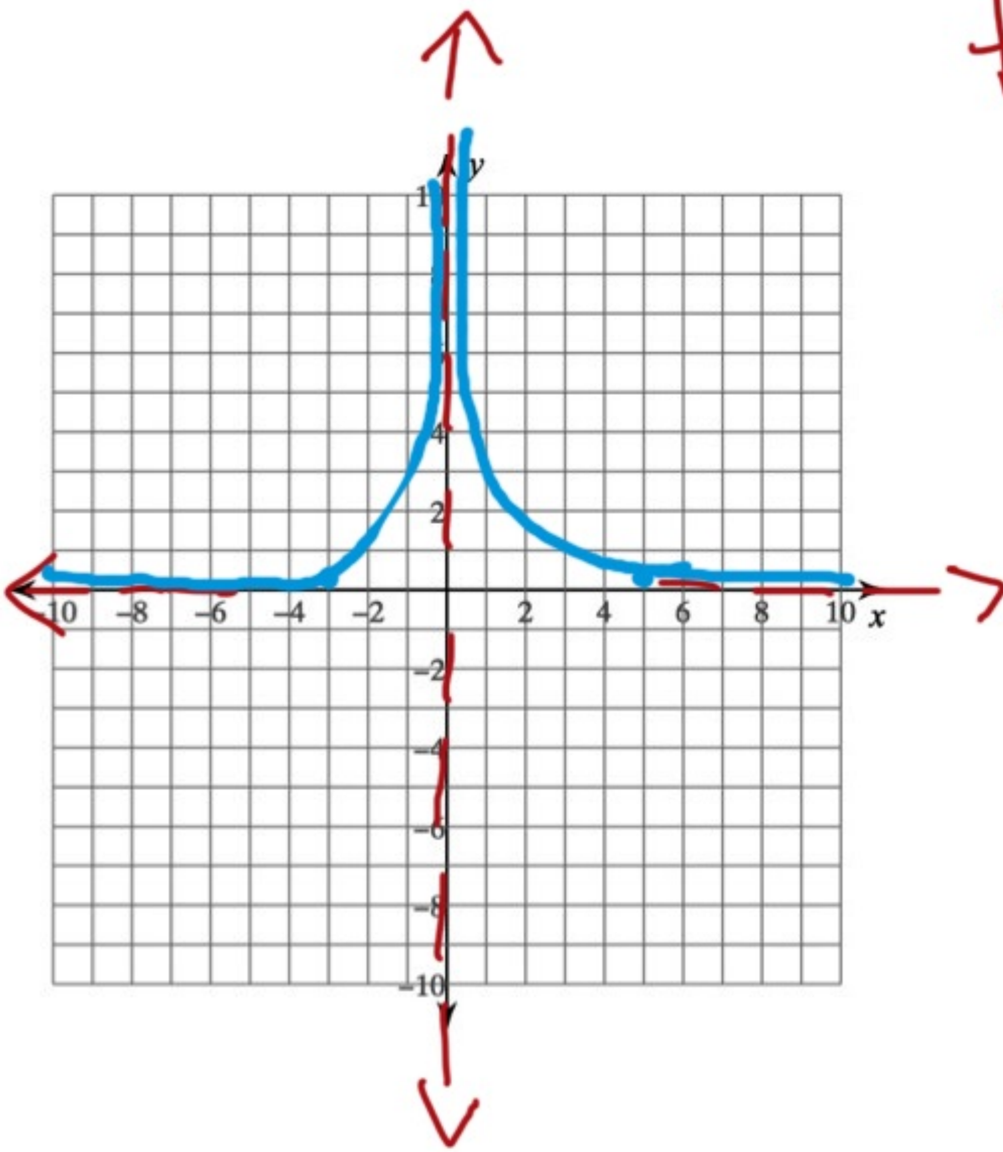
HA: (bottom heavy) $y = 0$

VA: $x = 0$ ⭑ Vertical asymptotes always outrank holes.

x-int: $4 \neq 0$ None

y-int: $\cancel{f(0) = \frac{4}{0^2}}$ None

B. Continued



$$f(5) = \frac{4}{5^2} = \frac{4}{25} = 0.16$$

$$f(-3) = \frac{4}{(-3)^2} = \frac{4}{9} = 0.44$$

C. $y = \frac{x+3}{x^2-6x+5} = \frac{x+3}{(x-1)(x-5)}, x \neq 1, 5$

non-removable

HA: $y=0$

VA: $x=1$, $x=5$

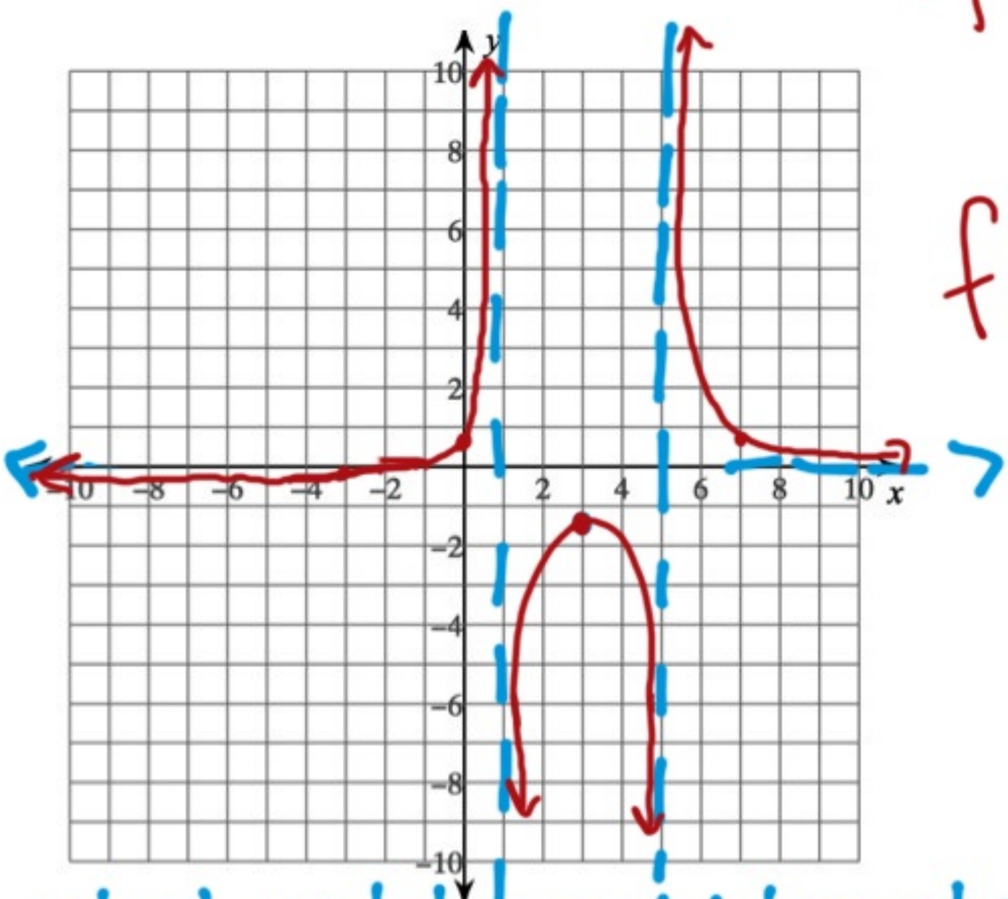
x-int: $x+3=0$

$(-3, 0)$

y-int: $f(0) = \frac{3}{5} = 0.6$

$(0, 0.6)$

C. continued



$$f(3) = \frac{3+3}{(3-1)(3-5)} = \frac{6}{-4} = -1.5$$

$$f(7) = \frac{7+3}{(7-1)(7-5)} = \frac{10}{12} = 0.83$$

* horizontal asymptotes only tell you <u>END BEHAVIOR</u>

→ you can cross in the middle.

D. $y = \frac{x^2+x}{x+1} = \frac{x\cancel{(x+1)}}{\cancel{x+1}} = x,\ x \neq -1 \rightarrow y = x$

HA: (top heavy) None

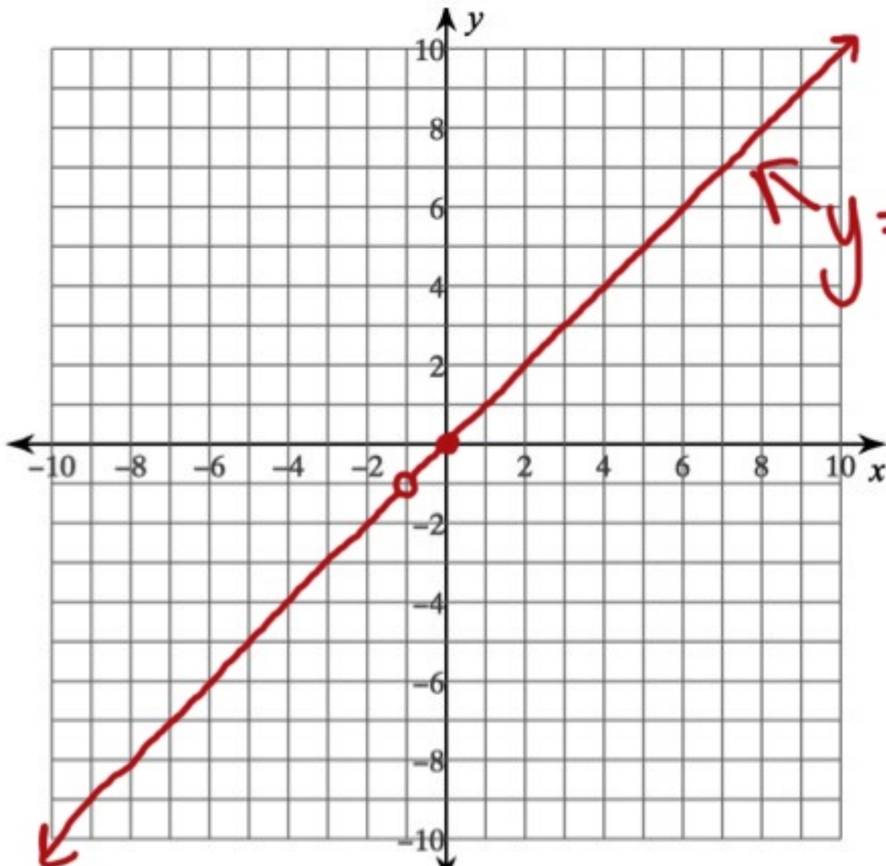
hole: $x=-1$

$f(-1) = -1 \quad (-1,-1)$

x-int: $x = 0 \quad (0,0)$

y-int: $(0,0)$

D. Continued



$y = x,\ x \neq -1$