

# 5.3: Solving Rational Equations

A. What are the solutions of the rational equation?

$$\frac{x}{x-3} + \frac{x}{x+3} = \frac{2}{x^2-9}$$

$$\frac{3}{3}x - \frac{3}{3} - 9$$

STEP 1: Find **domain** of all fractions (need to factor denominators)

STEP 2: Get LCD (least common denominator) by multiplying each fraction by the factor that it's missing. *\* same denominator*

STEP 3: Cancel denominators and solve.

STEP 4: Check solutions for domain.

$$\frac{(x+3) \cancel{x}}{(x+3)(x-3)} + \frac{(x-3) \cancel{x}}{(x-3)(x+3)} = \frac{2}{(x-3)(x+3)}$$

*\* set numerators equal*

$$x^2 + 3 \cancel{x} + x^2 - 3 \cancel{x} = 2$$

$$\frac{2x^2}{2} = \frac{2}{2}$$

$$\sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1} \checkmark$$

← Not an excluded value

$$\text{Domain: } (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

B.  $\frac{x-1}{x^2+3x+2} + \frac{2x}{x+2} = \frac{x-1}{x+1}$        $\frac{1 \times 2}{1+2} = \frac{2}{3}$       Domain:  $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

$$\frac{x-1}{(x+1)(x+2)} + \frac{\overbrace{2x(x+1)}^{2x(x+1)} \cdot \overbrace{(x-1)(x+2)}^{(x-1)(x+2)}}{\underbrace{(x+2)(x+1)}_{(x+1)(x+2)}} \quad (\text{common denominator})$$

$$x-1 + 2x^2 + 2x = x^2 + 2x - x - 2$$

$$\begin{array}{r} 2x^2 + 3x - 1 \\ -x^2 - x + 2 \\ \hline x^2 + 3x - 3 \end{array} = \begin{array}{r} x^2 + x - 2 \\ -x^2 - x + 2 \\ \hline 0 \end{array} \quad (\text{get everything on one side})$$

$$x^2 + 2x - 1 = 0 \quad \frac{1 \times 1}{1+1} = \frac{1}{2}$$

$$(x+1)^2 = 0$$

$$\rightarrow x = -1$$

Not in domain

**NO SOLUTION**

EXTRANEIOUS SOLUTION

D.  $\frac{x}{x+1} + \frac{3}{x+4} = \frac{x+3}{x+4}$

Domain:  $(-\infty, 4) \cup (-4, -1) \cup (-1, \infty)$

$$\frac{\overbrace{(x+4)}^{\cancel{x+4}} \cancel{x}}{\overbrace{(x+4)}^{\cancel{x+4}} (x+1)} + \frac{\overbrace{3}^{\cancel{x+1}} \overbrace{(x+1)}^{\cancel{x+1}}}{x+4} = \frac{\overbrace{(x+3)}^{\cancel{x+3}} \overbrace{(x+1)}^{\cancel{x+1}}}{\overbrace{(x+4)}^{\cancel{x+4}} \overbrace{(x+1)}^{\cancel{x+1}}}$$

$$x^2 + 4x + 3x + 3 = x^2 + 4x + 3$$

$$\begin{array}{r} x^2 + 7x + 3 \\ -x^2 - 4x - 3 \\ \hline 3x = 0 \end{array} = \begin{array}{r} x^2 + 4(x + 3) \\ -x^2 - 4x - 3 \\ \hline 3x = 0 \end{array}$$

$$\frac{3x}{3} = \frac{0}{3}$$

$x = 0$  ✓

G. You ride your bike to a store, 4 miles away, to pick up things for dinner. When there is no wind, you ride at 10 mi/hr. Today your trip to the store and back took you 1 hour. What was the speed of the wind today?

$$D = r \cdot t \quad t = \frac{D}{r}$$

	Distance	Rate	Time
With wind	4	$10 + x$	$\frac{4}{10+x}$
Against wind	4	$10 - x$	$\frac{4}{10-x}$

x: wind speed

$$\frac{\overbrace{4}^{(10-x)}}{\overbrace{(10-x)(10+x)}^{(10+x)}} + \frac{\overbrace{4}^{(10+x)}}{\overbrace{(10-x)(10+x)}^{(10-x)}} = \frac{1}{1} \frac{\overbrace{(10-x)(10+x)}^{(10-x)(10+x)}}{\overbrace{(10-x)(10+x)}^{(10-x)(10+x)}}$$

$D: (-\infty, -10) \cup (-10, 10) \cup (10, \infty)$

$$40 - \cancel{4x} + 40 + \cancel{4x} = 100 - x^2$$

$$80 = 100 - x^2$$

-100   -100

$$-20 = -x^2$$

$$\sqrt{x^2} = \sqrt{20}$$

$$x = 4.47 \text{ mph}$$

H. A boat goes 240 miles downstream in the same time it can go 160 miles upstream. The speed of the current is 5 miles per hour. What is the speed of the boat in still water?

	Distance	Rate	Time
With current	240	$X + 5$	$\frac{240}{X + 5}$
Against current	160	$X - 5$	$\frac{160}{X - 5}$

$x = \text{speed in still water}$  |  $O: (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

$$\frac{(x-5) 240}{(x-5)(x+5)} = \frac{160 (x+5)}{(x-5)(x+5)} \rightarrow 240x - 1200 = 160x + 800$$

$$-160x + 1200 - 160x + 1200$$

$$\frac{80}{80}x = \frac{2000}{80}$$

$$x = 25 \text{ mph}$$

J. You can wash one window in 15 minutes and your sister can wash one window in 20 minutes. How many minutes will it take to wash 12 windows if you work together?

$$\boxed{\text{You}} + \boxed{\text{Sister}} = \boxed{\text{Together}}$$
$$\frac{1 \text{ window}}{15 \text{ min}} + \frac{1 \text{ window}}{20 \text{ min}} = \frac{1 \text{ window}}{t \text{ min}}$$

$$\frac{4t}{4t} \frac{1}{15} + \frac{1}{20} \frac{3t}{3t} = \frac{1}{t} \frac{60}{60}$$

$$4t + 3t = 60$$
$$7t = 60$$

$$t = 60/7 \approx 8.57 \text{ min/window}$$

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$$\frac{12 \text{ windows}}{(8.57)12 = 102.88 \text{ min}}$$
$$\approx 1.71 \text{ hr}$$



K. One pump can fill a water tank twice as fast as a second pump. Working together, the two pumps can fill the cistern in 5 hours. Find how long it takes each pump to fill the cistern when working alone.

$$\boxed{\text{Pump 1}} \quad \frac{1 \text{ job}}{x \text{ hr}} \quad + \quad \boxed{\text{Pump 2}} \quad \frac{1 \text{ job}}{2x \text{ hr}} \quad = \quad \text{Together} \quad \frac{1 \text{ job}}{5 \text{ hr}}$$

$$\frac{10}{10} \frac{1}{x} + \frac{15}{2x5} = \frac{1}{5} \frac{2x}{2x}$$

$$\begin{aligned} 10 + 5 &= 2x \\ 15 &= 2x \\ x &= 7.5 \text{ hr} \end{aligned}$$

$$\boxed{\begin{aligned} \text{Pump 1} &: 7.5 \text{ hr} \\ \text{Pump 2} &: 15 \text{ hr} \end{aligned}}$$

1. On the first four tests of the term your average test score was 84%. You think you can score 96% on each of the remaining tests. How many consecutive test scores of 96% would you need to bring your average up to 90% for the term?

$x =$  sum of ~~1st~~ 4 tests

$$4 \cdot 84 = \frac{x}{4} \cdot 4$$

$$x = 336$$

$y =$  # of remaining test

$$\frac{(4+y)90}{4+y} = \frac{336 + 96y}{4+y}$$

$$\begin{array}{r} 360 + 90y = 336 + 96y \\ -336 - 90y \quad -336 \quad -90y \\ \hline 24 = 6y \end{array}$$

$$\frac{24}{6} = \frac{6y}{6}$$

$$\rightarrow \boxed{y = 4 \text{ tests}}$$