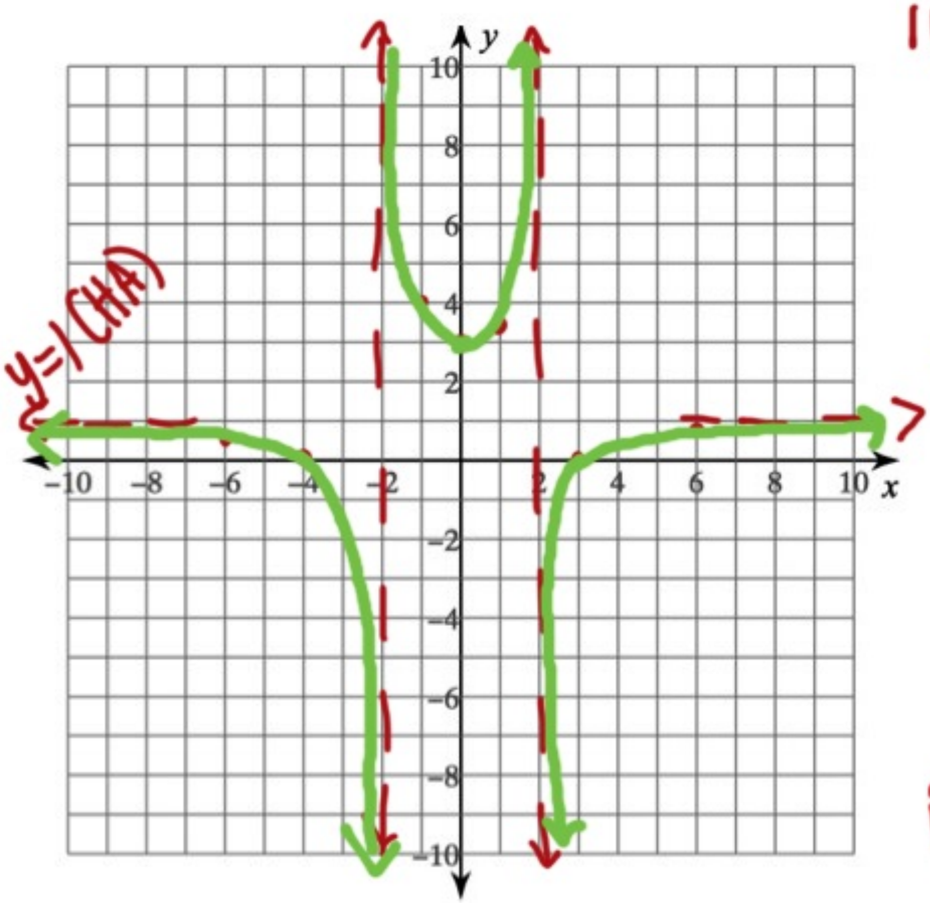
5.2 Graphing Rational Functions

You can get a reasonable graph for a rational function by finding all intercepts and asymptotes. Sometimes you will also have to plot a few extra points to get a good sense of the shape of the graph.

A. $y = \dfrac{x^2+x-12}{x^2-4}$

$\frac{4}{4}\; x \; \frac{-3}{-3} \; -12 \quad \frac{4}{4} + \frac{-3}{-3} \quad 1$

$\frac{2}{2}\; x \; \frac{-2}{-2} \; -4 \quad \frac{2}{2} + \frac{-2}{-2} \quad 0$



i) $y = 1$ (End Behavior)

ii) $\dfrac{(x+4)(x-3)}{(x+2)(x-2)}$, $x \neq -2, 2$

$\boxed{\text{VA: } x=2,\ x=-2}$

iii) x-int: $x+4=0$ $(-4,0)$

$x-3=0$ $(3,0)$

y-int: $f(0) = \frac{-12}{-4} = 3$

$(0,3)$

iv) $x=-6$: $(-6, 9/16)$ $x=6$: $(6, 15/16)$

$x=-1$: $(-1, 4)$

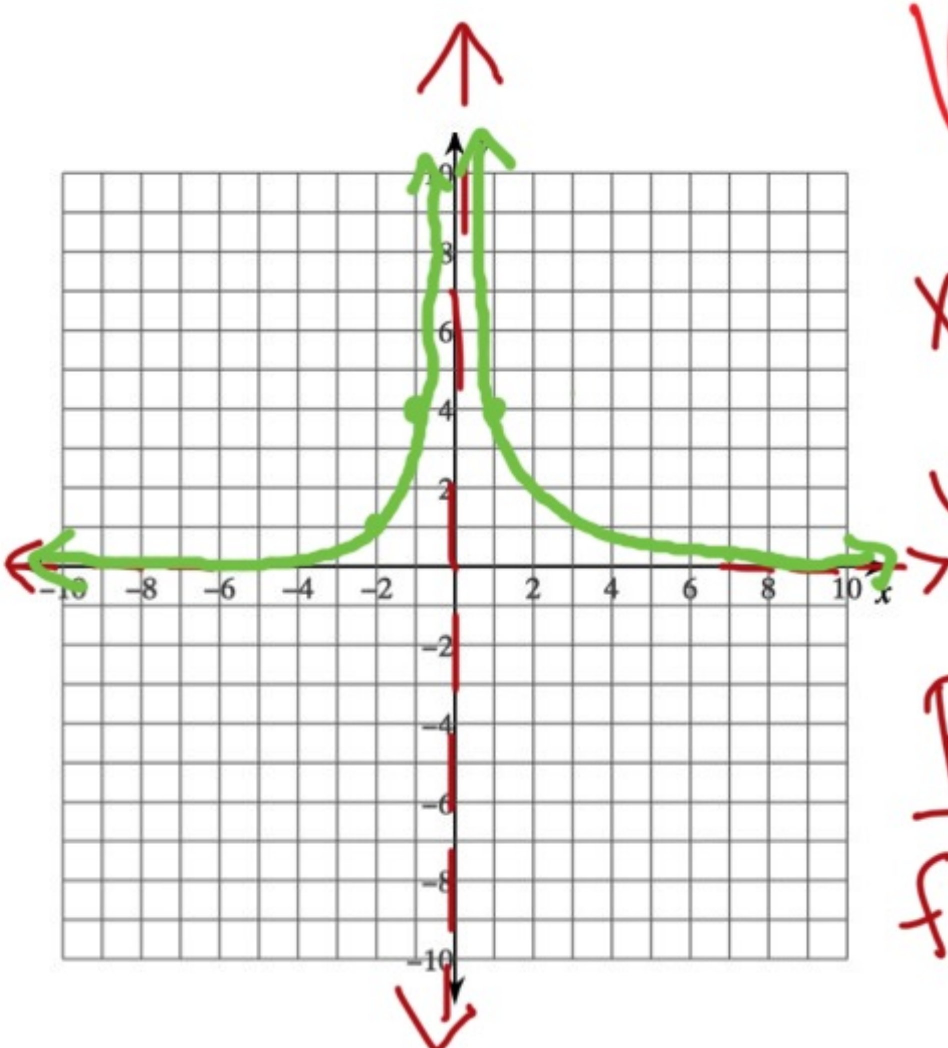
$x=1$: $(1, 10/3)$

B. $y = \frac{4x}{x^3} = \frac{4}{x^2}, x \neq 0$

HA: $y=0$

VA: $x=0$

* Vertical asymptote outranks a hole



x-int: $4 \neq 0$ NO x-int.

y-int: $\cancel{f(0) = \frac{4}{0^2}}$ No y-int

Plug in

$f(1) = \frac{4}{1^2} = 4$ $f(-2) = \frac{4}{(-2)^2} = \frac{4}{4} = 1$

$f(-1) = \frac{4}{(-1)^2} = 4$ $f(7) = \frac{4}{7^2} = \frac{4}{49}$

C. $y = \frac{x+3}{x^2-6x+5} = \frac{x+3}{(x-1)(x-5)}$

$-1 \times -5 = 5$
$-1 + -5 = -6$

$x \neq 1, 5$

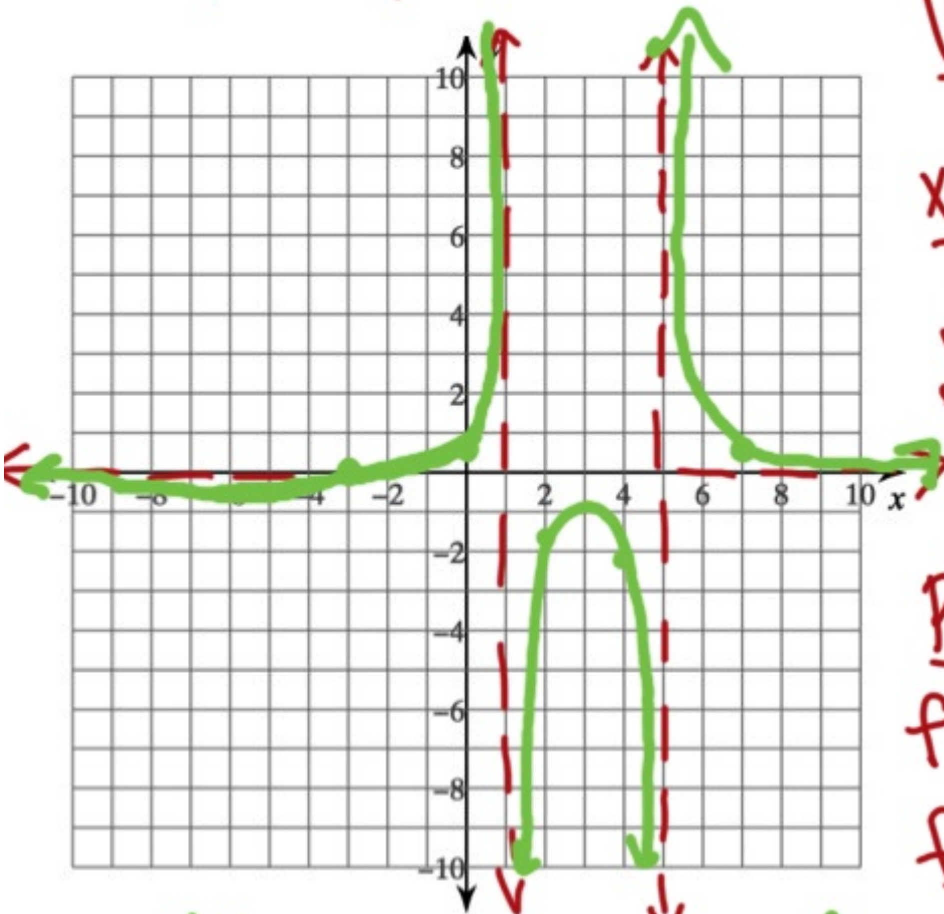
HA: $y = 0$

VA: $x = 1, x = 5$

x-int: $x+3=0$ $(-3,0)$

y-int: $f(0) = \frac{3}{5} = 0.6$

$(0, 0.6)$



Plug in

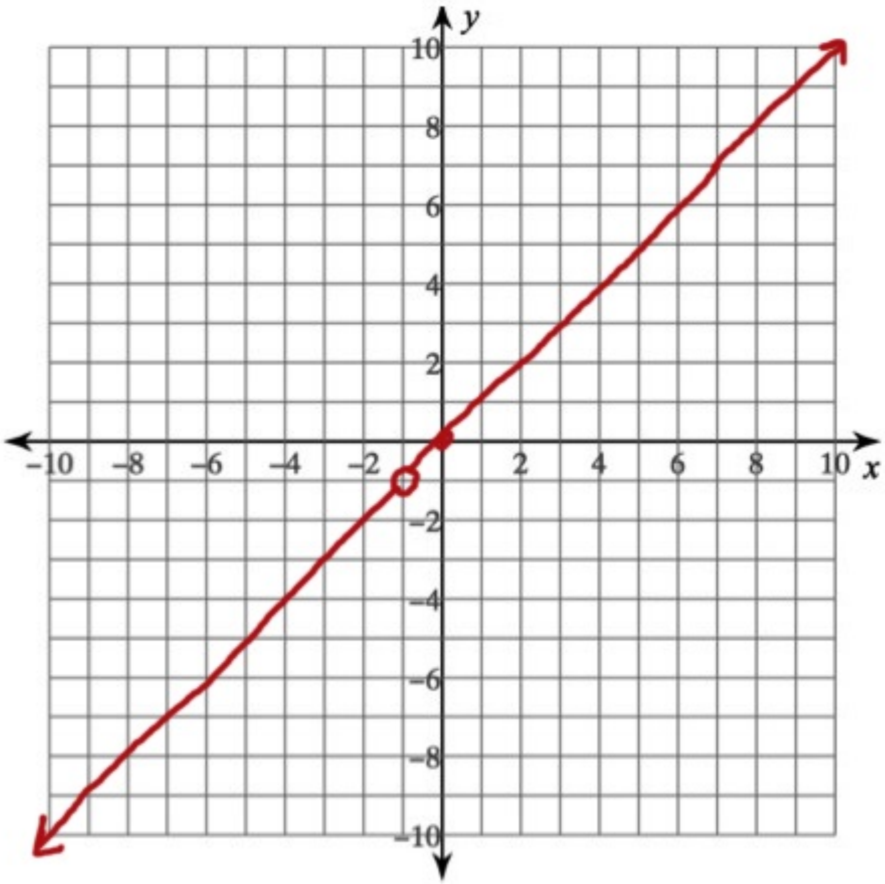
$f(2) = \frac{2+3}{(2-1)(2-5)} = \frac{5}{-3}$

$f(7) = \frac{7+3}{(7-1)(7-5)} = \frac{10}{12} = \frac{5}{6}$

$f(4) = \frac{4+3}{(4-1)(4-5)} = \frac{7}{-3}$

* HA are behavioral asymptotes, so you can cross the asymptote in the middle.

D. $y = \frac{x^2+x}{x+1} = \frac{x(\cancel{x+1})}{\cancel{x+1}} = x, \; x \neq -1$



HA: None

VA: None

Holes: $x = -1$

$f(-1) = -1$

$\boxed{(-1,-1)}$

X-int: $x = 0$

$(0,0)$

y-int: $(0,0)$